HW1 MTH 450-550 Fall 2023

(p.8) $\boxed{\#1}$ $\{x \in \mathbb{R} \mid x^2 = 3\} = \{\sqrt{3}, -\sqrt{3}\}$

#2) $\{m \in \mathbb{Z} \mid m^2 = 3\} = \emptyset$

#6) $\{n \in \mathbb{Z} \mid n^2 < 0\}$

The set is well-defined because it is easy to check if any given integer is in there.

It happens to also be the empty set!

#7) $\{n \in \mathbb{Z} \mid 39 < n^3 < 57\}$

well-defined, happens to be empty!

#8) $\{x \in \mathbb{Q} \mid x \text{ is almost an integer}\}$

Not well-defined because what "almost an integer" means is not clear.

#9) $\{x \in \mathbb{Q} \mid x \text{ may be written with a denominator} > 100\}$

well defined!

It actually is all of $\mathbb{Q}$ (why?)

#16) List power sets:

(a) $\emptyset \rightsquigarrow \mathcal{P}(\emptyset) = \{\emptyset\}$ and $|\mathcal{P}(\emptyset)| = 1$

(b) $\{a\} \to \mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$ and $|\mathcal{P}(\{a\})| = 2$

(c) $\{a,b\} \to \mathcal{P}(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$ and $|\mathcal{P}(\{a,b\})| = 4$

(d) $\{a,b,c\} \to \mathcal{P}(\{a,b,c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ and $|\mathcal{P}(\{a,b,c\})| = 8$

#18) $B^A \overset{\text{def}}{:=} \{f : A \to B\}$ with $B = \{0,1\}$

Claim: $|B^A| = |\mathcal{P}(A)|$

We can associate any function $f : A \to \{0,1\}$ to a subset S of A given by

$S = \{a \in A : f(a) = 1\}$.

Vice versa any subset $T \subseteq A$ can be associated to some ~~func~~ function $g : A \to B$ where $g(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$

So each function and subset are paired up!

This defines a bijection between $\mathcal{P}(A)$ and B^A, completing the proof.