

Written HW4 – MATH 4580 Spring 2023

Let X be a set. A topology τ on X is a collection of subsets of X that obeys three properties:

- $\emptyset \in \tau$ and $X \in \tau$,
- any union of elements of τ is in τ , and
- any intersection of finitely many elements of τ is in τ .

If τ is a topology on X , then we say that each set in τ is an open set.

In this homework, we will explore this (strange) definition of a topology.

1. Let $X = \{a, b\}$ and let $\tau = \{\emptyset, \{a, b\}, \{a\}\}$. Is τ a topology on X ? Why or why not? If not, then what needs to be added to τ to make it a topology?

2. How many topologies exist for the set $X = \{a, b\}$? List them all. (*hint: start with listing all subsets of X*)

3. Let $X = \{a, b, c\}$ and let $\tau = \{\emptyset, \{a, b, c\}, \{a\}, \{b\}\}$. Is τ a topology on X ? Why or why not? If not, then what needs to be added to τ to make it a topology?