Quiz 8 – MATH 2510 Spring 2023

Recall the axioms of the theory of first order arithmetic:

1. $(\forall x)(0 \neq Sx)$
2. $(\forall x)(\forall y)(Sx = Sy \rightarrow x = y)$
3. $(\forall y)(y = 0 \vee (\exists x)(Sx = y))$
4. $(\forall x)(x + 0 = x)$
5. $(\forall x)(\forall y)(x + Sy = S(x + y))$
6. $(\forall x)(x \cdot 0 = 0)$
7. $(\forall x)(\forall y)(x \cdot Sy = (x \cdot y) + x)$
8. $(\forall x)(\forall y)(x + y = y + x)$
9. $(\forall x)(\forall y)(x \cdot y = y \cdot x)$

Prove that $a + 0 = 0 \longleftrightarrow a = 0$ is a theorem of 1st order arithmetic.