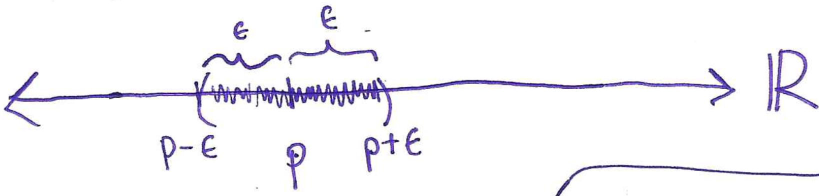
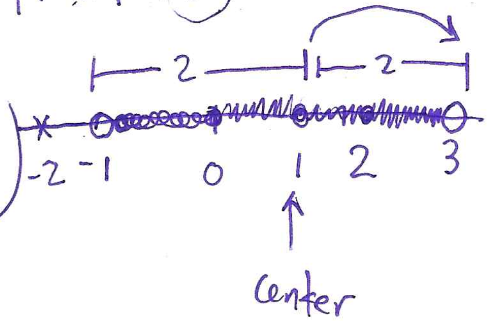
"centered at 1" → $|x-1| = |1-x| < 2$

$|1-(-2)| = |3| = 3$

Let $\epsilon > 0$.



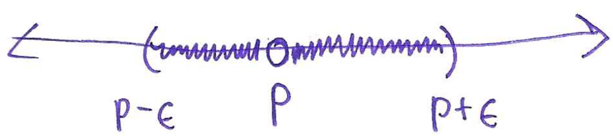


$$N_\epsilon(p) = \{x \in \mathbb{R} : |x-p| < \epsilon\}$$

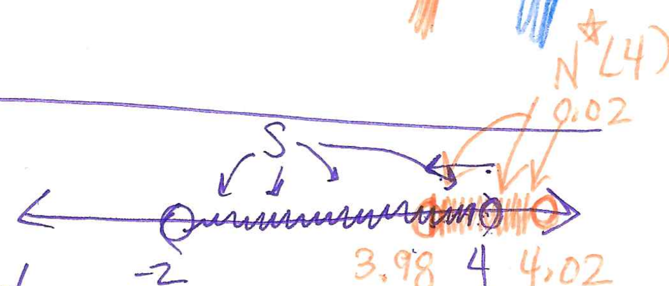
$$= (p-\epsilon, p+\epsilon)$$

$$N^*_\epsilon(p) = N_\epsilon(p) \setminus \{p\}$$

→ will be used in limits



Ex 2.5 $S = (-2, 4)$



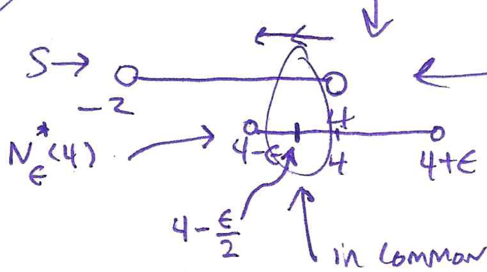
Show $\boxed{N^*_{0.02}(4)} \cap S \neq \emptyset$.

$N^*_{0.02}(4) = (4-0.02, 4+0.02)$

$(3.98, \cancel{3}4.02)$

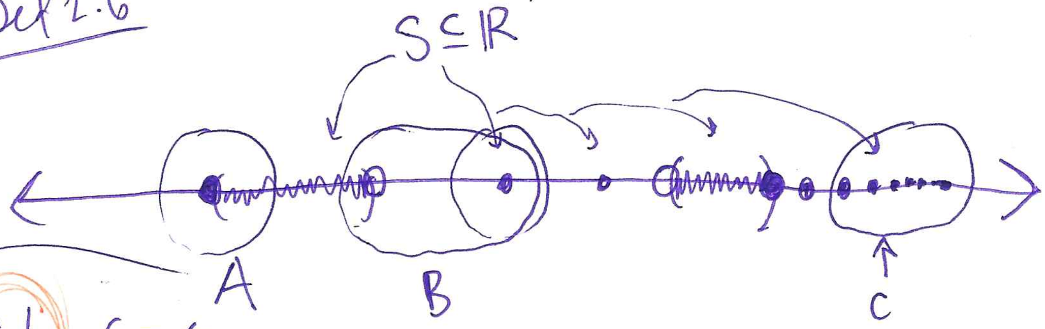
for example, $3.99 \in N^*_{0.02}(4) \cap S$

Also $\boxed{N^*_\epsilon(4) \cap S \neq \emptyset}$ for any $\epsilon > 0$.



$4 - \frac{\epsilon}{2} \in N^*_\epsilon(4) \cap S$

Def 2.6

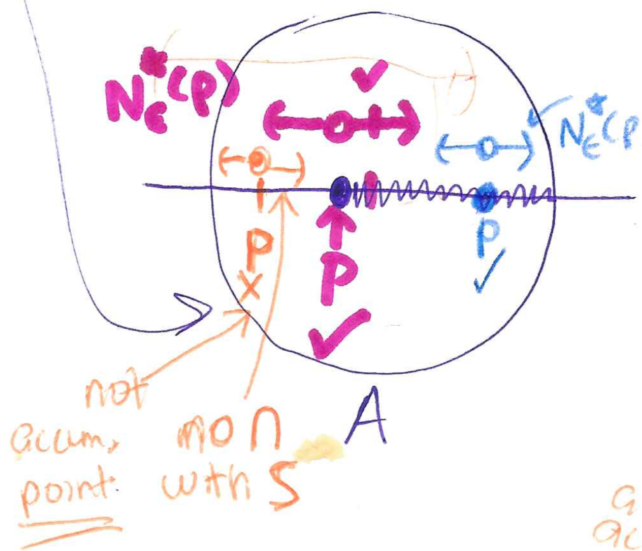


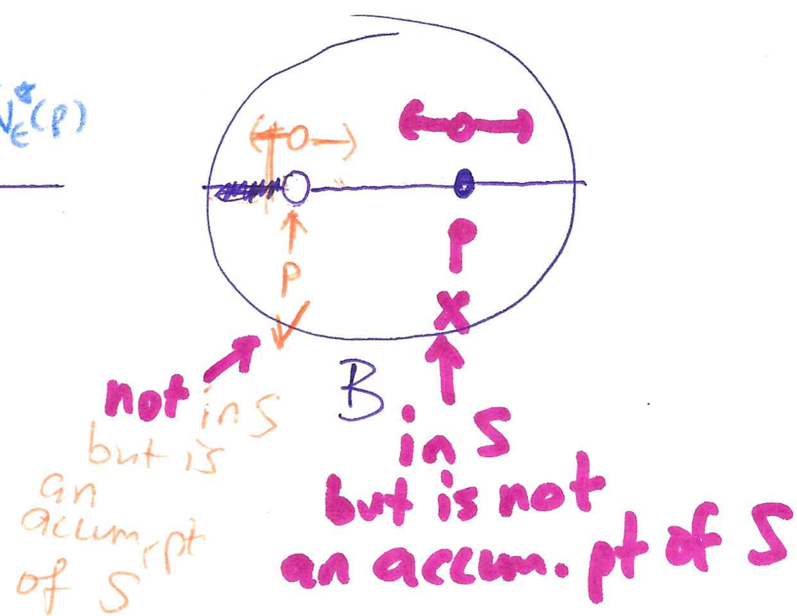
$\forall\ \epsilon > 0$

$S \cap N^*_\epsilon(p) \neq \emptyset$

$S' = \{x \in \mathbb{R} : x \text{ is accumulation pt of } S\}$

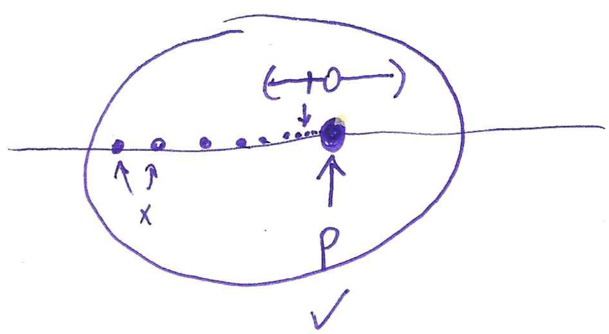
"Cantor-Bendixson derivative"





$$\left\{1-\frac{1}{n} : n \in \mathbb{N}\right\} \quad \left(\frac{n-1}{n}\right)$$

$= \{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots\} \cup \{1\}$

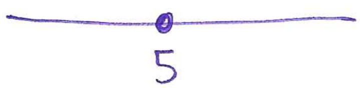


$S \subseteq \mathbb{R}$

Def 2.10 $\quad cl(S) = S \cup S'$

Def 2.11 S <u>closed</u> means $S = cl(S)$

EX: $S = \{5\}$



S has no accumulation pts

So $S' = \emptyset$

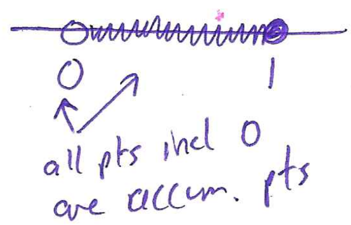
$cl(S) = S \cup S' = S \cup \emptyset = S$

$\Rightarrow S$ is closed

$\overline{[a, b]}$

"closed interval"

EX: $S = (0, 1]$



$S' = [0, 1]$

$cl(S) = S \cup S'$

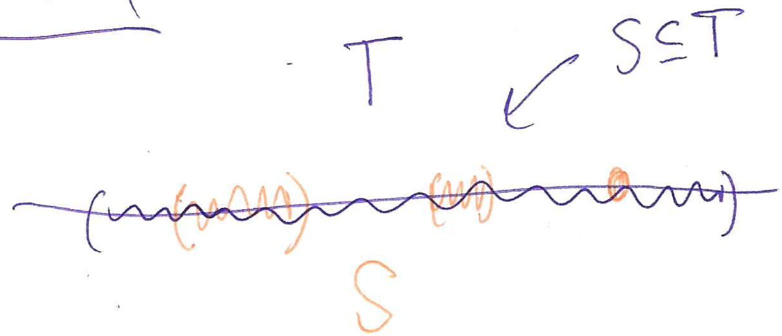
$= (0,1] \cup [0,1]$

$= [0,1] \neq S$

$\Rightarrow S$ is not closed

Def 2.29

$$S \subseteq T \land T \subseteq S \Rightarrow T = S$$



S is dense in T means $T \subseteq cl(S)$

$S \subseteq T \land T \subseteq cl(S) \Rightarrow$ S is dense in T

Ex: $T = [1,2]$

$S = (1,2)$

$S \subseteq T$ ✓ $\quad S' = [1,2]$

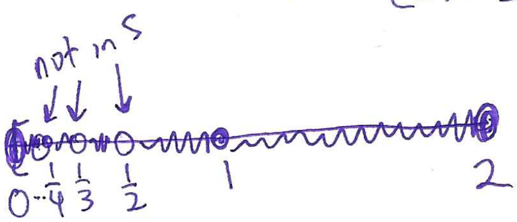
$cl(S) = S \cup S' = [1,2]$

Therfore $T = [1,2] \subseteq [1,2] = S$

$\Rightarrow$ S is dense in T

Ex: $T = [0,2]$

$S = [0,2] \setminus \{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$



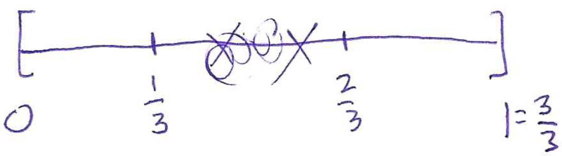
$S' = [0,2]$

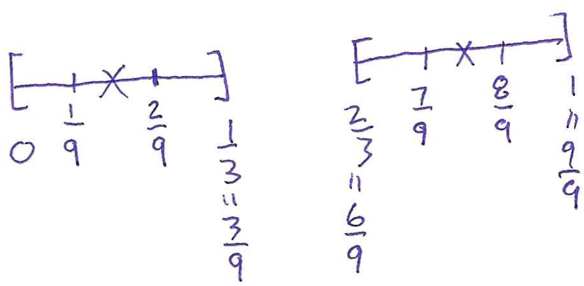
$cl(S) = S \cup S' = [0,2]$

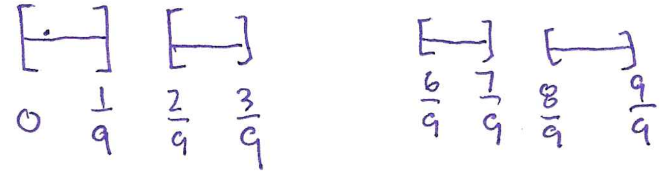
$T = [0,2] \subseteq [0,2] = cl(S)$

$\Rightarrow$ S is dense in T

Def 2.32

$C_0 =$ 

$C_1 = C_0 \setminus \left(\frac{1}{3}, \frac{2}{3}\right)$ 

$C_2 =$ 

C_{n+1} formed by deleting middle $\frac{1}{3}$'s from C_n sets

Def 2.37

Cantor set

$$C_\infty = \bigcap_{k=1}^{\infty} C_k \leftarrow \text{nonempty!}$$

e.g. $0 \in C_\infty$
$1 \in C_\infty$
$\frac{7}{9} \in C_\infty$
etc