

Written HW10 – MATH 2502 Spring 2022

Due by 23 February for timely completion credit

Consider the so-called “lower incomplete gamma function”

$$\gamma_s(x) = \int_0^x t^{s-1} e^{-t} dt. \quad (1)$$

is a function of two variables “ s ” and “ x ”.

This function has published applications in numerous areas of science such as protein folding in biology, “Gaussian orbitals” in quantum chemistry, and ecological systems theory (see <https://dlmf.nist.gov/8.24>). It is related to the so-called “gamma function” Γ which is one of the most important functions in mathematics (see https://en.wikipedia.org/wiki/Gamma_function).

- #1. Write the integral corresponding to $\gamma_1(x)$ by setting $s = 1$ in (1).
- #2. Solve the integral you found in #1.
- #3. Find $\gamma_2(x)$ using integration by parts. Your answer will be a function of the variable x .
- #4. Use Desmos to plot the functions $\gamma_1(x)$, $\gamma_2(x)$, $\gamma_3(x)$, $\gamma_4(x)$, and $\gamma_5(x)$ on the same plot. You can use the integral formulation to accomplish this. (*note: you were asked to do something similar in WHW9 #4*).
- #5. Perform integration by parts on the integral (1) by letting $u = t^{s-1}$ and $dv = e^{-t}$. Be careful not to use “ x ” as a variable except as a bound of the integral or being plugged in for t when appropriate!!
- #6. Note that in #5, the “ $s - 1$ ” you should have gotten that appears in the integral is not dependent on t – it is a constant with respect to t (no different than something like “7”) – so pull it out of the integral like you would any other constant.
- #7. Now you should have

$$\gamma_s(x) = \text{“a function of } x\text{”} + (s - 1)\text{“some kind of integral”}. \quad (2)$$

Which lower incomplete gamma function does the integral in that expression correspond to (*in other words, what should “ s ” be replaced by in (1) to obtain what you’ve got here?*)? Rewrite the expression (2) by filling in the “some kind of integral” expressed as the correct lower incomplete gamma function and filling in the “function of x ” part you discovered in #5. This expression is known as the “recurrence relation” associated to the lower incomplete gamma function and is fundamental to its use.