Written HW5 – MATH 3504 Spring 2021
Due by 15 February for timely completion credit

In class we derived the spring-mass system differential equation

$$mx'' + \gamma x' + kx = 0, \qquad (1)$$

where m is the mass of the spring, γ is the damping coefficient, and k is the spring constant.

The solution function $x(t)$ describes the distance that the spring moves **away from** its equilibrium (i.e. non-moving) position) – so when the output of the function x is zero, we think of the spring as being "at equilibrium". Indeed, you can check that the function $x(t) = 0$ is a solution of the differential equation (1)! We will equip this differential equation with two intitial conditions: $x(0) = x_0$ (corresponding to the initial stretching of the system – i.e. how far from equilibrium you stretch the mass before you release) and $x'(0) = x_1$ (corresponding to the initial velocity of the system – i.e. what velocity **you** impart to the mass before releasing it *(think "throwing" it vs "letting go of it")*).

In this homework, you will solve such equations (the eigenvalues will become distinct real roots here).

1. Solve (1) when $m = 1$, $\gamma = 3$, and $k = 2$ with initial conditions $x(0) = 0$ and $x'(0) = 1$. Describe what the initial conditions mean in this case. What happens to the solution as $t \to \infty$ and what does this mean about this spring system?

2. Solve (1) when $m = 1$, $\gamma = 6$, and $k = 9$ with initial conditions $x(0) = 1$ and $x'(0) = 0$. Describe what the initial conditions physically mean about the system in this case. What happens to the solution as $t \to \infty$ and what does this mean about this spring system?