Ex: $RI' + \frac{1}{C} I = E'(t)$ $\quad \boxed{x' + p(t) x = q(t)}$

Find current in circuit when $R = 2$

and $C = 1$ and $E(t) = \sin(t)$

w/ init cond $I(0) = 0$.

Soln: $2I' + I = \cos(t)$

$$I' + \underbrace{\left(\tfrac{1}{2}\right)}_{p(t)} I = \frac{\cos(t)}{2}$$

integrating factor: $\mu(t) = e^{\int p(t)\,dt} = e^{\frac{1}{2}t}$

$$\left(I e^{\frac{1}{2}t}\right)' = \frac{e^{\frac{1}{2}t}\cos(t)}{2}$$

$$I e^{\frac{1}{2}t} = \underbrace{\int \frac{e^{\frac{1}{2}t}\cos(t)}{2}\,dt}_{\text{int by parts twice}}$$

$$I e^{\frac{1}{2}t} = \frac{2}{5} e^{\frac{1}{2}t}\left(2\sin(t) + \cos(t)\right) + C$$

$$I(t) = \frac{2}{5}\left(2\sin(t) + \cos(t)\right) + C e^{-\frac{1}{2}t}$$

$$\underbrace{0}_{\text{given}} = I(0) = \frac{2}{5}\left(2\overset{0}{\cancel{\sin(0)}} + \overset{1}{\cancel{\cos(0)}}\right) + C e^{0}$$

$$0 = \frac{2}{5} + C \rightarrow C = -\frac{2}{5}$$

$$I(t) = \left(\frac{2}{5}\right)\left(2\sin(t) + \cos(t)\right) - \frac{2}{5} e^{-\frac{1}{2}t}$$

Ch. 2 2nd order ODE's

$x' = F(t,x)$

§2.1 Classical Mechanics

$$\left[\begin{array}{l} x'' = 5 \to x' = 5t + C \\ \qquad\qquad \to x = \frac{5}{2}t^2 + Ct + D \end{array}\right]$$

Newton's 2nd Law: force = mass · accel

Let $x(t)$ be position function

[Know calc 1: $x'(t) \sim$ velocity

$x''(t) \sim$ accel]

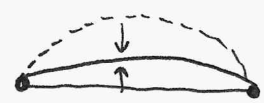
$$\boxed{\begin{cases} m \cdot x''(t) = F(t, x, x') \\ x(0) = x_0,\ x'(0) = x_1 \end{cases}}$$

Alternative

Give "boundary conditions"

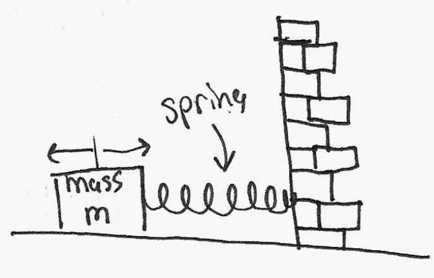
$x(0) = x_0$

$x(5) = x_1$

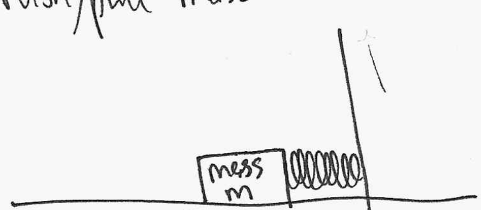


§2.1.1 Oscillations + Dissipation

Example 2.1: (Oscillator)



Push/pull mass



let go

By Newton's law...

$$mx'' = F_s \leftarrow F_s \text{ is the force given by the spring}$$

Experiments show

$$F_s = -kx \leftarrow \text{(Hooke's law)}$$

for some $k>0$ ↑ (units $\frac{\text{force}}{\text{distance}}$)

minus b.c. spring force is opposite the motion

k called "spring constant"

Initial cond: $x(0) = x_0 \leftarrow$ position released

$x'(0) = x_1 \leftarrow$ velocity given at time $t=0$ ("let go" $\sim x_1 = 0$)

"Guess" a soln of form

$$\begin{cases} x = A\cos(\omega t) \\ \Rightarrow x' = -\omega A \sin(\omega t) \\ \Rightarrow x'' = -\omega^2 A\cos(\omega t) \end{cases}$$

Plug in to $mx'' = -kx$

$$\Downarrow$$

$$-m\omega^2 \cancel{A\cos(\omega t)} = -k\cancel{A\cos(\omega t)}$$

$$-m\omega^2 = -k$$

$$\omega = \pm\sqrt{\frac{k}{m}}$$

cosine even

$$\Rightarrow x(t) = A\cos\left(\pm\sqrt{\frac{k}{m}}t\right) = A\cos\left(\sqrt{\frac{k}{m}}t\right)$$

$\boxed{\text{What is A? } \quad x_0 = x(0) = A}$

Other IC?

$0 = x'(0) = -\sqrt{\frac{k}{m}}\, x_0 \sin(0)$

$0 = 0$... useless info!!

$$\Rightarrow \boxed{x(t) = x_0\cos\left(\sqrt{\frac{k}{m}}t\right)}$$

$$-m\omega^2 \cancel{A\cos(\omega t)} = -k\cancel{A\cos(\omega t)}$$ (4)

$$-m\omega^2 = -k$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \pm\sqrt{\frac{k}{m}}$$

related to frequency of cos

spring const + mass

$$\Rightarrow \quad x(t) = A\cos\left(\pm\sqrt{\frac{k}{m}}\,t\right)$$

cosine even

$$= A\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

Apply IC's

$$x_0 = x(0) = A\overset{1}{\cancel{\cos}}(0)$$

$$\boxed{A = x_0}$$

$$0 = x'(0) = -\omega x_0 \overset{0}{\cancel{\sin}}(0)$$

$$\underbrace{0=0}_{\text{useless}}$$

$\Rightarrow$ We obtain

$$x''(t) = x_0\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

cos function with period

$$\text{per} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

Our previous model has no friction

Friction "dissipates" energy in system

Ex: (Damped oscillator)

$$mx'' = F$$

$$F_s = -kx \qquad F_d = -\gamma x'$$

damping coeff

$$F = F_s + F_d$$

$\Rightarrow$ Damped oscillator eqt:

$$mx'' = -\gamma x' - kx$$

can solve

$$x(t) = Ae^{-\lambda t}\cos(\omega t)$$

Mechanical - Electrical Analogy

SAME?!

$$\begin{cases} m x'' + \gamma x' + k x = 0 \\ L I'' + R I' + \frac{1}{C} I = 0 \end{cases}$$

~ damped oscillator

"inertia"

"resistance to change"

"dissipate energy"

"storage of energy"