2021.01.15

§1.1.2

$x(t)$

decay rate

$$x' = -rx, \quad r > 0$$

$\downarrow$ div by x

$$\boxed{\frac{x'}{x} = -r}$$

Consider

$$\frac{d}{dt}\ln\big(x(t)\big) = \frac{x'(t)}{x(t)}$$

$$\frac{d}{dt}\ln\big(x(t)\big) = -r$$

$\downarrow$ $\int \dots dt$

$$\ln(x(t)) = -\int r\,dt$$

$$= -rt + C$$

$\downarrow$ plug both sides into e^x

$$e^{a+b} = e^a e^b$$

$$\cancel{e}^{\ln(x(t))} = e^{-rt+C}$$

$$x(t) = C_1 e^{-rt} \; ; \; C_1 = e^C$$

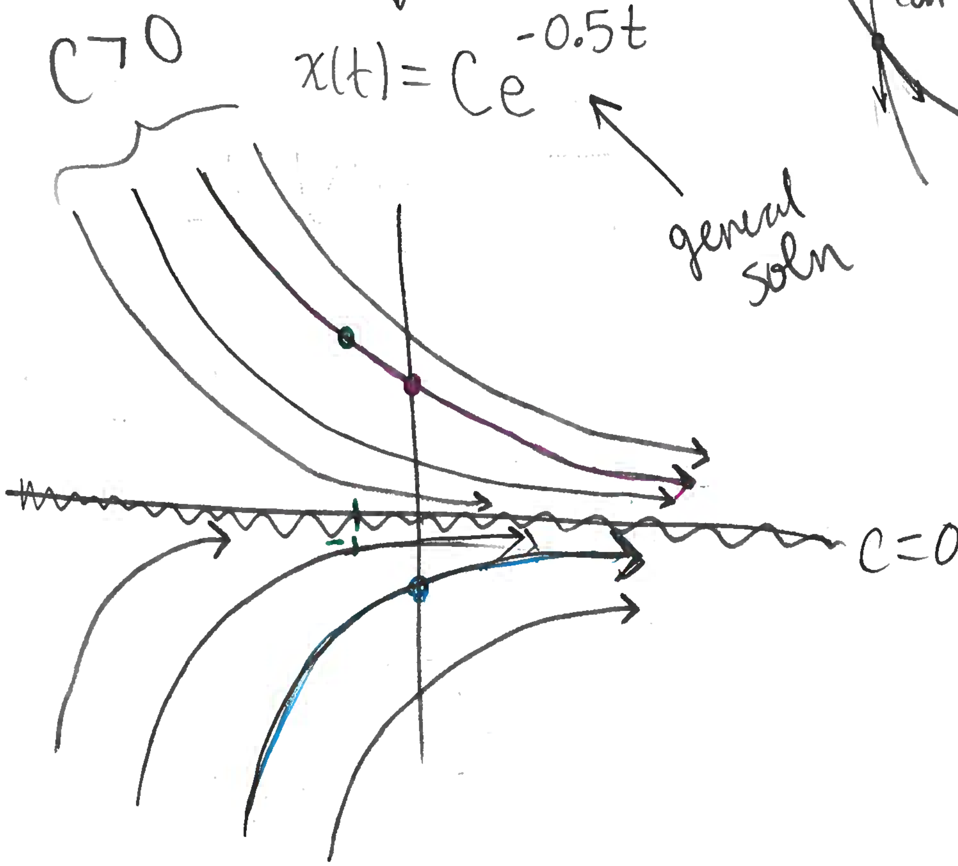
arbitrary constant

Example: $x' = -0.5x$

$(r = 0.5)$

$$x(t) = Ce^{-0.5t}$$

general soln



equip w/ initial value

$x(0) = 1$ $\quad$ $x(-1) = e^{0.5}$

$x(0) = -1$

Ex 1.3 6 grams of radioactive material decays to 1.3g in 5 years

What is r? What is the half-life?

the time it takes for half of the given material to decay

Soln: The DE is

$$x' = -rx, \quad r>0$$

$\downarrow$ solve

$$x(t) = Ce^{-rt}$$

measures grams at time t (years)

Let t=0 be start time. So initial condition is

$$x(0)=6$$

$\downarrow$ Apply

$$\underset{\uparrow \text{given}}{6} = x(0) = \underbrace{Ce^{0}}_{\text{calculated from our soln}}$$

$\downarrow$

$$\boxed{C=6}$$

$\downarrow$ update

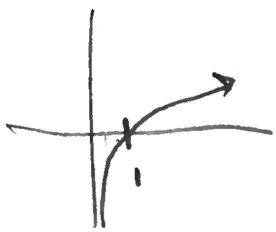
$$x(t) = 6e^{-rt}$$

The info that it decays to $\underbrace{1.3}_{x(t)\ \text{value}}$ in $\underbrace{5\ \text{years}}_{t\text{-value}}$

$\Downarrow$

$$\boxed{x(5) = 1.3}$$

$\Downarrow$ apply to our function



$$\underbrace{1.3}_{\text{given}} = x(5) = \underbrace{6e^{-5r}}_{\text{computed}}$$

$\downarrow$

$$0.306 \approx \frac{\ln\left(\frac{1.3}{6}\right)}{-5} = r$$

$\Downarrow$ update

$$x(t) = 6e^{-0.306t}$$

$\Downarrow$ find half life

Find time t_ℓ such that

$$3 = x(t_\ell) = 6e^{-0.306t_\ell}$$

$$2.265 \approx \frac{\ln\left(\frac{3}{6}\right)}{-0.306} = t_\ell$$

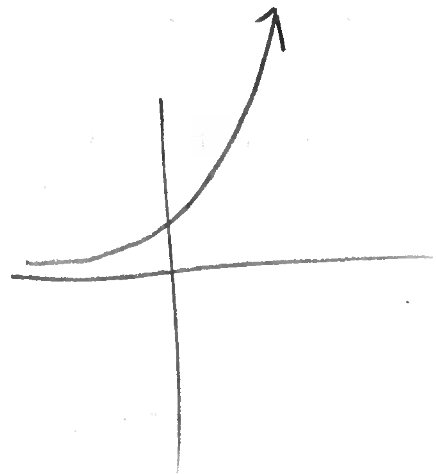
Growth model

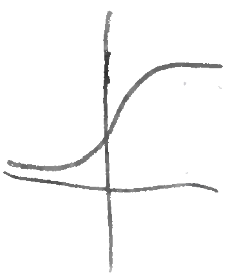
$$x' = rx \,, \quad r > 0$$

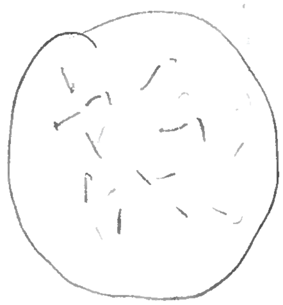
Population growth

("Malthusian")

$$\boxed{x(t) = Ce^{rt}}$$





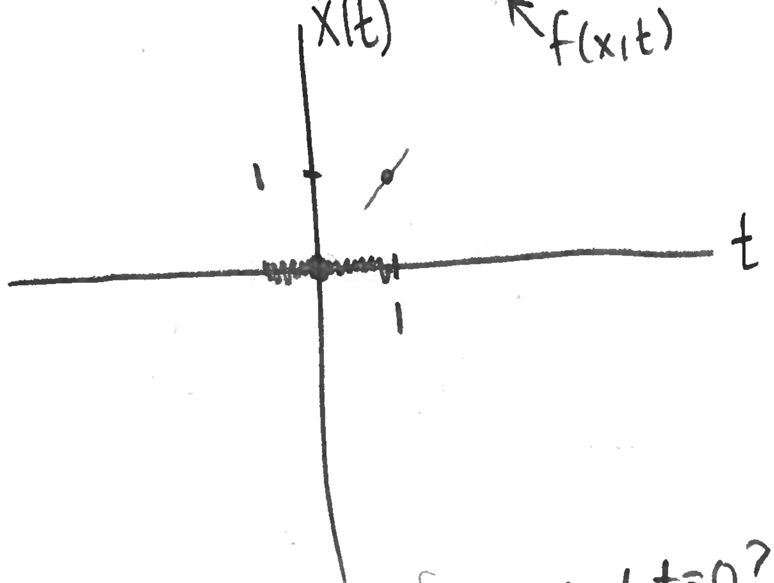


Geometric approach

$$\text{"} \; x' = f(t,x) \; \text{"}$$

slope is given by this

Ex 1.5 $x' = \underbrace{-\overset{x(t)}{x} + 2t}_{f(x,t)}$



What is slope when... $x=0$ and $t=0$?

$x' = f(0,0) = 0$

$x=1,\ t=1$

$x' = f(1,1) = -1+2 = 1$

Use computer