Written HW17 – MATH 3503 Fall 2021
Due by 22 October for timely completion credit

Recall that Green's theorem says that if D is a simply-connected region, its boundary C is parametrized with positive orientation (counter-clockwise), and $\vec{F} = \langle P, Q\rangle$, then

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \mathrm{d}A.$$

Green's theorem allows for the computation of a double integral as a line integral AND the computation of a line integral as a double integral. Below you will see examples of both.

1. Set up ***but do not evaluate*** both sides of Green's theorem for the following scenario: D is the region bounded by the graphs of $y = x$ and $y = \sqrt{x}$ and $\vec{F} = \langle xe^y, e^x\rangle$.

2. Use Green's theorem to calculate the line integral

$$\int_C (y - x)\mathrm{d}x + (2x - y)\mathrm{d}y,$$

   where C is the boundary curve of the region lying between the graphs of $y = x$ and $y = x^2 - 2x$

3. We will use Green's theorem to calculate the double integral

$$\iint_D y - x\mathrm{d}A,$$

   where D is the unit disk (so its boundary is the unit circle). (*hint: the hard part is finding an $\vec{F}$ that works! in other words, you need to pick P and Q so that $\vec{F} = \langle P, Q\rangle$ and $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = y - x$, but also allows you to calculate the line integral...keep it simple!*)