

p.302

#1

no  
premise

$$\vdash \forall x F(x) \rightarrow \forall y (F(y) \wedge F(y))$$

1	$\forall x F(x)$	
2	$F(a)$	$\forall E 1$
3	$F(a) \wedge F(a)$	$\wedge I 2,2$
4	$\forall y (F(y) \wedge F(y))$	$\forall I 3$
5	$\forall x (F(x) \rightarrow \forall y (F(y) \wedge F(y)))$	$\rightarrow I 1-4$

#2]  ~~$\forall x (A(x) \rightarrow B(x)), \exists x A(x) \vdash \exists x B(x)$~~

1	$\forall x (A(x) \rightarrow B(x))$	
2	$\exists x A(x)$	
3	$A(a)$	$\exists E 2$
4	$A(a) \rightarrow B(a)$	$\forall E 1$
5	$B(a)$	$\rightarrow E 4,3$
6	$\exists x B(x)$	$\exists I 5$

↑  
bad application  
of  $\exists E$

fixed on p.3

illegal

ok → 3	$A(a) \rightarrow B(a)$	$\forall E 1$
4	$A(a)$	$\exists E 2$

illegal'

can't reuse name "a"  
with  $\exists E$

↑  
moral: do  $\exists$  first  
and  $\forall$  second, when  
possible!

#5  $\vdash \forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$   
 no premises

1	$\forall x R(x,x)$	
2	$R(a,a)$	$\forall E 1$
3	$\exists y R(a,y)$	$\exists I 2$
4	$\exists x \exists y R(x,y)$	$\exists I 3$
5	$\forall x R(x,x) \rightarrow \exists x \exists y R(x,y)$	$\rightarrow I 1-4$

#8  $\forall x \forall y (G(x,y) \rightarrow G(y,x)) \vdash \forall x \forall y (G(x,y) \leftrightarrow G(y,x))$

	1	$\forall x \forall y (G(x,y) \rightarrow G(y,x))$	
	2	$G(a,b)$	
11	3	$\forall y (G(a,y) \rightarrow G(y,a))$	$\forall E 1$
12	4	$G(a,b) \rightarrow G(b,a)$	$\forall E 3$
	5	$G(b,a)$	$\rightarrow E 4, 2$
	6	$G(b,a)$	
	7	$\forall y (G(b,y) \rightarrow G(y,b))$	$\forall E 1$
	8	$G(b,a) \rightarrow G(a,b)$	$\forall E 7$
	9	$G(a,b)$	$\rightarrow E 8, 6$
	10	$G(a,b) \leftrightarrow G(b,a)$	$\leftrightarrow I 2-5, 6-9$

$G(a,b) \leftrightarrow G(b,a)$

$\forall I 10$  (points to line 11)  
 $\forall I 11$  (points to line 12)

#2

$\forall x (A(x) \rightarrow B(x)), \exists x A(x) \vdash \exists x B(x)$

3

1  $\forall x (A(x) \rightarrow B(x))$

2  $\exists x A(x)$

3  $A(a)$

4  $A(a) \rightarrow B(a)$   $\forall E$  1

5  $B(a)$   $\rightarrow E$  4, 3

6  $\exists x B(x)$   $\exists I$  5

7  $\exists x B(x)$   $\exists E$  2, 3-6

p.313 B2 Show provably equivalent:

$\forall x (A(x) \rightarrow \neg B(x))$  and  $\neg \exists x (A(x) \wedge B(x))$

$\forall x (A(x) \rightarrow \neg B(x)) \vdash \neg \exists x (A(x) \wedge B(x))$       $\neg \exists x (A(x) \wedge B(x)) \vdash \forall x (A(x) \rightarrow \neg B(x))$

1  $\forall x (A(x) \rightarrow \neg B(x))$   
 2  $\exists x (A(x) \wedge B(x))$   
 3  $A(a) \wedge B(a)$   
 4  $A(a)$   $\wedge E$  3  
 5  $B(a)$   $\wedge E$  3  
 6  $A(a) \rightarrow \neg B(a)$   $\forall E$  1  
 7  $\neg B(a)$   $\rightarrow E$  6,4  
 8  $\perp$   $\neg E$  7,5  
 9  $\perp$   $\exists E$  2,3-8  
 10  $\neg \exists x (A(x) \wedge B(x))$   $\neg I$  2-9

1  $\neg \exists x (A(x) \wedge B(x))$   
 2  $\forall x \neg (A(x) \wedge B(x))$   $CQ$  1  
 3  $\neg (A(a) \wedge B(a))$   $\forall E$  2  
 4  $A(a)$   
 5  $B(a)$   
 6  $A(a) \wedge B(a)$   $\wedge I$  4,5  
 7  $\perp$   $\neg E$  3,6  
 8  $\neg B(a)$   $\neg I$  5-7  
 9  $A(a) \rightarrow \neg B(a)$   $\rightarrow I$  4-8  
 10  $\forall x (A(x) \rightarrow \neg B(x))$   $\forall I$  9