Show

(*) $\exists x\, A(x,x) \to B(d)$

is not a validity

Sufficient to find an interpretation where (*) is false.

interpretation: domain: 1, 2

A(x,y):

tells me
A(1,1) true
A(1,2), A(2,1), A(2,2) are all false

$\begin{cases} B(x) : \_\_ x = 1 \\ d : 2 \end{cases}$

B(d) means "2 = 1" which is false!

In this interpretation,

$\exists x\, A(x,x)$ is true since letting $x = 1$ yields $A(1,1)$ which is true

BUT

$B(d)$ is false

So we conclude

$$\underbrace{\exists x\, A(x,x)}_{\text{true}} \to \underbrace{B(d)}_{\text{false}}$$

is false

Show $\exists x\, A(x,x) \to B(d)$ $(\star)$

is not a contradiction.

Suffices to show an interpretation which makes $(\star)$ true.

<u>interpretation:</u> domain: 1, 2

$\boxed{A(x,y): \text{false always}}$

$B(x)$: $x = 1$

d: 2

In this interpretation,

$\exists x\, A(x,x)$ is false

And $B(d)$ is false

Consequently:

$$\underbrace{\underbrace{\exists x\, A(x,x)}_{\text{false}} \to \underbrace{B(d)}_{\text{false}}}_{\text{true since "false} \to \text{anything" is always true!}}$$

Show $\forall x\, S(x)$ and $\exists x\, S(x)$

are not logically equivalent.

<u>interpretation:</u>

domain: 1, 2

$S(x)$: $\underline{\quad}_{x} = 1$

In this interpretation:

$\forall x\, S(x)$ is false because if $x = 2$, then $S(2)$ is false

$\exists x\, S(x)$ is true because if $x = 1$, then $S(1)$ is true

Consider

$$\exists x\,(G(x) \to G(a)) \therefore \exists x\, G(x) \to G(a)$$

We seek to show this argument is invalid, meaning there is an interpretation where

$$\exists x\,(G(x) \to G(a)) \text{ is true}$$

and

$$\exists x\, G(x) \to G(a) \text{ is false}$$

<u>interpretation</u>: domain: 1, 2

G(x): G(1) true, G(2) false

a: 2

In this interpretation:

$$\exists x\,(G(x) \to G(a)) \text{ true because } x=2 \text{ then}$$

$$\underbrace{\underbrace{G(2)}_{\text{false}} \to \underbrace{G(a)}_{\text{false}}}_{\text{true}}$$

BUT

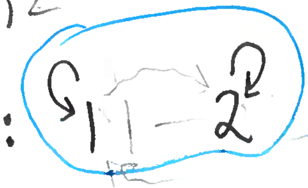
$$\underbrace{\underbrace{\underbrace{\exists x\, G(x)}_{x=1}}_{\text{true}} \to \underbrace{G(a)}_{\text{false}}}_{\text{false}}$$

Show

$$\forall x \exists y\, L(x,y) \therefore \exists y \forall x\, L(x,y)$$

is invalid.

interpretation: domain: 1, 2

$L(x,y)$: 

tells us that
$L(1,1)$ true
$L(2,2)$ true
$L(1,2)$ false
$L(2,1)$ false

In this interpretation:

$\forall x \exists y\, L(x,y)$ is true!

Because...

if $x=1$ $\downarrow$ take $y=1$ $\downarrow$ $L(x,y)$ true

and

if $x=2$ $\downarrow$ take $y=2$ $\downarrow$ $L(x,y)$ true

BUT

$\exists y \forall x\, L(x,y)$ is false!

Because...

if $y=1$ $\downarrow$ but if $x=2$ $L(2,1)$ false

OR

if $y=2$ $\downarrow$ take $x=1$ and $L(1,2)$ is false

p. 280

$$G \quad \begin{tikzcd} 1 \arrow[loop left] \arrow[r] \arrow[dr] & 2 \arrow[d] \\ & 3 \end{tikzcd}$$

$\exists x(\exists y_1 R(y_1,x) \wedge \neg\exists y_2 R(x,y_2))$ ~ supposedly true...

Can $x=1$?

$y_1 = 1$ ✓

no y_2 works because 1 points to everything

but "$\neg\exists y_2\, R(x,y_2)$" says that x does not point to y_2

✗

Can $x=2$? ✓

$y_1 = 1$ ✓

take $y_2 = 1$: because $\neg R(2,1)$ is true since no arrow from 2 to 1

✓

yes ~ see that the sentence is true