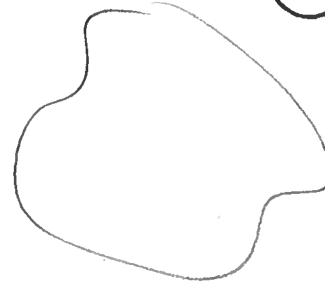
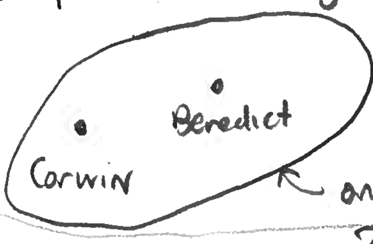


$\forall x\, P(x)$

has no truth value unless an interpretation is chosen!!

interpretation: domain, domain object associated to all names used, and T/F for all domain objects for all predicates used

p. 271 A | interpretation is given...



domain

only 2 objects!

(*) $A(x)$ true of both Carwin & Benedict

(*) $B(x)$ true of only Benedict

(*) $N(x)$ always false

(*) c ~ Carwin

TRUE or FALSE in our interp.?

1. $B(c)$ ~ false b/c B only true for Benedict

2. $A(c) \leftrightarrow \neg N(c)$ ~ true
   $A(c)$ true ✓
   $\neg N(c)$ true ✓

3. $N(c) \rightarrow (A(c) \vee B(c))$ ~ true because "False → Anything" is true

4. $\forall x\, A(x)$ ~ true because A is true of everything in domain

5. $\forall x\, \neg B(x)$ ~ false b/c $B(b)$ is true when b is Benedict, so $\neg B(b)$ is false
   occurs in interp.
   $I[\text{Benedict}/b]$

6. $\exists x(A(x) \land B(x))$ ~ true b/c in interp. $I[\text{Benedict}/b]$

$A(b) \land B(b)$ is true!

7. $\exists x(A(x) \to N(x))$ ~ false A is always true

N is always false

and "True → False" is false

8. $\forall x(N(x) \lor \neg N(x))$ ~ true because $N(x)$ ~ always false

$\neg N(x)$ ~ always true

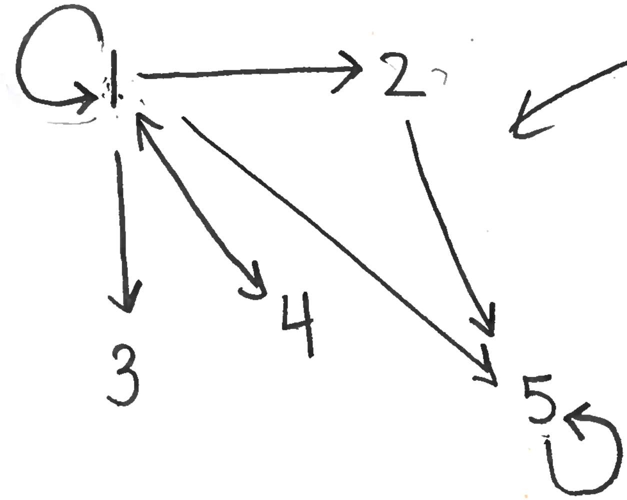
so False ∨ True is true

9. $(\exists x\, B(x)) \to (\forall x\, A(x))$ ~ true because

$\exists x\, B(x)$ is true (look at $I[\text{Benedict}/b]$)

and $\forall x A(x)$ is true since A is always true

C (p.272)



this tells us which R(a,b) are true

#1) $\exists x\, R(x,x)$ ~ true for example, $R(1,1)$ is true

#4) $\exists x \forall y\, R(y,x)$ ~ false this sentence encodes the notion that there is a domain object "x" such that all objects "y" have an arrow to x — "3" has no arrow leaving it at all...!

Can x=5 and y=5?
yes BUT
all y's must be considered

#3) $\exists x \forall y\, R(x,y)$ ~ true because IN $I[1/\theta]$
$\forall y(R(\theta,y))$ is true

#6) $\forall x \forall y \forall z\,((R(x,y) \wedge R(x,z)) \rightarrow R(y,z))$ ~ false ~ it fails here

$x=1, y=2, z=5$
$\downarrow$
$(\underbrace{R(1,2) \wedge R(1,5)}_{\text{true}}) \rightarrow \underbrace{R(2,5)}_{\text{true}}$ ✓

$x=1, y=3, z=4$
$\downarrow$
$\left.(\underbrace{R(1,3) \wedge R(1,4)}_{\text{true}}) \rightarrow \underbrace{R(3,4)}_{\text{false}}\right\}$ false

#5 $\forall x \forall y \forall z \left((R(x,y) \land R(y,z)) \to R(x,z) \right)$ ~ false

because if

$x=4,\ y=1,\ z=5$ then

$$\underbrace{\underbrace{(\underbrace{R(4,1)}_{\text{true}} \land \underbrace{R(1,5)}_{\text{true}})}_{\text{true}} \to \underbrace{R(4,5)}_{\text{false}}}_{\text{"true} \to \text{false" is false}}$$

#10 $\exists x \forall y \left(R(x,y) \leftrightarrow x=y \right)$ ~ true (when $x=5$)

$x=5$

$\to$ $y=1$

$$\underbrace{\underbrace{R(5,1)}_{\text{false}} \leftrightarrow \underbrace{5=1}_{\text{false}}}_{\text{true}}$$

$y=2$

$$\underbrace{\underbrace{R(5,2)}_{F} \leftrightarrow \underbrace{5=2}_{F}}_{T}$$

$y=3,4$ ~ same stuff

$y=5$

$$\underbrace{\underbrace{R(5,5)}_{\text{true}} \leftrightarrow \underbrace{5=5}_{\text{true}}}_{\text{true}}$$