Prove $A \lor B, \neg B \therefore A$

Without rule DS:

1	$A \lor B$	
2	$\neg B$	
3	$\quad A$	
4	$\quad A$	R3
5	$\quad B$	
6	$\quad \bot$	$\neg$E 2,5
7	$\quad A$	X 6
8	A	$\lor$E 1, 3-4, 5-7

With rule DS:

1	$A \lor B$	
2	$\neg B$	
3	A	DS 1,2

Ex: $P \therefore (P \land D) \lor (P \land \neg D)$

1	P		
2	$\quad D$		lot of freedom
3	$\quad P \land D$	$\land I$ 1,2	
4	$\quad (P \land D) \lor (P \land \neg D)$	$\lor I$ 3	
5	$\quad \neg D$		
6	$\quad P \land \neg D$	$\land I$ 1,5	
7	$\quad (P \land D) \lor (P \land \neg D)$	$\lor I$ 6	
8	$(P \land D) \lor (P \land \neg D)$	LEM 2-4, 5-7	

B1

p.168

$E \vee F, F \vee G, \neg F \therefore E \wedge G$

1	$E \vee F$	
2	$F \vee G$	
3	$\neg F$	
4	E	DS 1,3
5	G	DS 2,3
6	$E \wedge G$	$\wedge$I 4,5

p.160
B4

$(X \wedge Y) \vee (X \wedge Z),\ \neg(X \wedge D),\ D \vee M \therefore M$

1	$(X \wedge Y) \vee (X \wedge Z)$	
2	$\neg(X \wedge D)$	
3	$D \vee M$	
4	$\neg X \vee \neg D$	DeM 2
5	$X \wedge Y$	
6	X	$\wedge$E 5
7	$\neg X$	
8	$\bot$	$\neg$E 7,6
9	$\neg\neg X$	$\neg$I 7–8
10	$\neg D$	DS 4,9
11	M	DS 3,10
12	$X \wedge Z$	
13	X	$\wedge$E 12
14	$\neg X$	
15	$\bot$	$\neg$E 14,13
16	$\neg\neg X$	$\neg$I 14–15
17	$\neg D$	DS 4,16
18	M	DS 3,17
19	M	$\vee$E 1, 4-11, 12-18

semantics

(study of "truth" as a concept)

$\mathcal{A}_1, \dots, \mathcal{A}_n \vDash \mathcal{C}$

means the argument with premises $\mathcal{A}_1, \dots, \mathcal{A}_n$ ~~and~~ and conclusion $\mathcal{C}$ is a $\underbrace{\text{valid argument}}_{\text{"no counterexample"}}$

syntax

(grammar, rules of proof, proofs themselves, etc)

$\mathcal{A}_1, \dots, \mathcal{A}_n \vdash \mathcal{C}$

means there is a proof w/ premises $\mathcal{A}_1, \dots, \mathcal{A}_n$ that ends with $\mathcal{C}$

if $\mathcal{A}_1, \dots, \mathcal{A}_n \vDash \mathcal{C}$ then does $\mathcal{A}_1, \dots, \mathcal{A}_n \vdash \mathcal{C}$?

→ if $\mathcal{A}_1, \dots, \mathcal{A}_n \vdash \mathcal{C}$ then does $\mathcal{A}_1, \dots, \mathcal{A}_n \vDash \mathcal{C}$?

In TFL, yes both work!

In FOL : can formalize

"I am not provable."

(work of K. Gödel in 1940's)

$\vDash \mathcal{A}$ means that $\mathcal{A}$ is a tautology

$\vdash \mathcal{A}$ means we can prove $\mathcal{A}$ with no premises

Ex: Show that $\vdash \neg(A \wedge \neg A)$ (ie. show $\neg(A \wedge \neg A)$ is a theorem)

1	$\quad A \wedge \neg A$	
2	$\quad A$	$\wedge E$ 1
3	$\quad \neg A$	$\wedge E$ 1
4	$\quad \bot$	$\neg E$ 3,2
5	$\neg(A \wedge \neg A)$	$\neg I$ 1–4

Ex p.171 A1 Show $\vdash \theta \to \theta$

1	$\quad \theta$	
2	$\quad \theta$	R1
3	$\theta \to \theta$	$\to I$ 1-2