

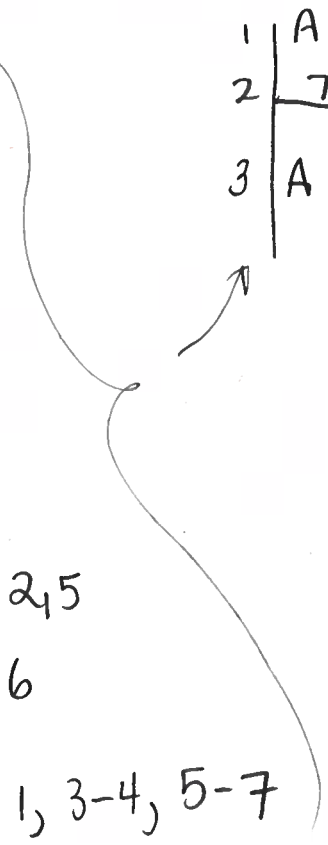
Prove  $A \vee B, \neg B \therefore A$

Without rule DS:

1		$A \vee B$	
2		$\neg B$	
<hr/>			
3		$A$	
<hr/>			
4		$A$	R3
<hr/>			
5		$B$	
<hr/>			
6		$\perp$	$\neg E$ 2,5
<hr/>			
7		$A$	X 6
<hr/>			
8		$A$	$\vee E$ 1, 3-4, 5-7

With rule DS:

1		$A \vee B$	
2		$\neg B$	
<hr/>			
3		$A$	DS 1,2



Ex: P: (P∧D) ∨ (P∧¬D)

	1	P	
lot of freedom	2	D	
	3	P∧D	∧I 1,2
	4	(P∧D) ∨ (P∧¬D)	∨I 3
	5	¬D	
	6	P∧¬D	∧I 1,5
	7	(P∧D) ∨ (P∧¬D)	∨I 6
	8	(P∧D) ∨ (P∧¬D)	LEM 2-4, 5-7

(B1)

P.169

$E \vee F, F \vee G, \neg F \therefore E \wedge G$

(3)

1	$E \vee F$	
2	$F \vee G$	
3	$\neg F$	
<hr/>		
4	E	DS 1, 3
5	G	DS 2, 3
6	$E \wedge G$	AI 4, 5

p. 160

B4

4

$(X \wedge Y) \vee (X \wedge Z), \neg(X \wedge D), DVM \therefore M$

1	$(X \wedge Y) \vee (X \wedge Z)$	
2	$\neg(X \wedge D)$	
3	DVM	
<hr/>		
→ 4	$\neg X \vee \neg D$	DeM 2
5	$X \wedge Y$	
6	$X$	$\wedge E$ 5
7	$\neg X$	
8	$\perp$	$\neg E$ 7, 6
9	$\neg \neg X$	$\neg I$ 7-8
10	$\neg D$	DS 4, 9
11	M	DS 3, 10
12	$X \wedge Z$	
13	$X$	$\wedge E$ 12
14	$\neg X$	
15	$\perp$	$\neg E$ 14, 13
16	$\neg \neg X$	$\neg I$ 14-15
17	$\neg D$	DS 4, 16

18	M	DS 3, 17
19	M	VE 1, 4-11, 12-18

Semantics  
 study of  
 ("truth" as a  
 concept)

Syntax  
 (grammar, rules of proof,  
 proofs themselves, etc)

$$A_1, \dots, A_n \models B$$

means the argument  
 with premises  $A_1, \dots, A_n$   
~~and~~ and conclusion  $B$  is  
 a valid argument

"no counterexample"

$$A_1, \dots, A_n \vdash B$$

means there is a proof  
 w/ premises  $A_1, \dots, A_n$  that  
 ends with  $B$

→ if  $A_1, \dots, A_n \models B$  then does  $A_1, \dots, A_n \vdash B$ ?  
 → if  $A_1, \dots, A_n \vdash B$  then does  $A_1, \dots, A_n \models B$ ?

In TFL, yes both work!

In FOL : can formalize

"I am not provable."

(work of K. Gödel in 1940's)

$\models A$   
means that  
 $A$  is a tautology

$\vdash A$   
means we can prove  
 $A$  with no premises

Ex: Show that  $\vdash \neg(A \wedge \neg A)$  (ie. show  $\neg(A \wedge \neg A)$  is a theorem)

1	$A \wedge \neg A$	
2	$A$	$\wedge E 1$
3	$\neg A$	$\wedge E 1$
4	$\perp$	$\neg E 3, 2$
5	$\neg(A \wedge \neg A)$	$\neg I 1-4$

Exp. 171 A1 Show  $\vdash \emptyset \rightarrow \emptyset$

1	$\emptyset$	
2	$\emptyset$	$R1$
3	$\emptyset \rightarrow \emptyset$	$\rightarrow I 1-2$