

Show $\neg(A \wedge B)$ is equivalent to $(\neg A) \vee (\neg B)$.

(1)

| A | B | $\neg(A \wedge B)$ | $(\neg A) \vee (\neg B)$ |
|---|---|--------------------|--------------------------|
| T | T | F | F |
| T | F | T | T |
| F | T | T | T |
| F | F | T | T |

SAME \rightarrow equivalent!

Ex: Show $\neg(G \rightarrow M)$ entails G.

| G | M | $\neg(G \rightarrow M)$ | G | $G \rightarrow M$ |
|---|---|-------------------------|---|-------------------|
| T | T | F | T | F |
| T | F | T | T | T |
| F | T | F | F | F |
| F | F | F | F | T |

not equivalent

p.90

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| P | Q | P | $P \rightarrow Q$ | Q |
|---|---|---|-------------------|---|
| T | T | T | T | T |
| T | F | T | F | F |
| F | T | F | T | T |
| F | F | F | T | F |

premises conclusion

No valuation where both P and $P \rightarrow Q$ are true and simultaneously Q false.

Ex: $\models PV \supset P$

| P | $PV \supset P$ |
|---|----------------|
| T | T |
| F | T |

\models ~ semantics thing
living in English
(metalinguage)

\rightarrow - connective in TFL ~
syntax thing

p.92 B2 | jointly satisf. or j-unsatisf. ?

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$A \vee B, A \rightarrow C, B \rightarrow C$

| A | B | C | $A \vee B$ | $A \rightarrow C$ | $B \rightarrow C$ |
|---|---|---|------------|-------------------|-------------------|
| T | T | T | T | T | T |
| T | T | F | T | F | F |
| T | F | T | T | T | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | T | F | T | T | F |
| F | F | T | F | T | T |
| F | F | F | F | T | T |

this row is a valuation in which all the sentences are true!

↓
conclude these are jointly satisfiable

p.92 B3 | j. satisf. or j. unsatisf?

$B \wedge (C \vee A), A \rightarrow B, \neg(B \vee C)$

| A | B | C | $B \wedge (C \vee A)$ | $A \rightarrow B$ | $\neg(B \vee C)$ |
|---|---|---|-----------------------|-------------------|------------------|
| T | T | T | T | T | F |
| T | T | F | T | T | F |
| T | F | T | F | F | F |
| T | F | F | F | F | T |
| F | T | T | T | T | F |
| F | T | F | F | T | F |
| F | F | T | F | T | F |
| F | F | F | F | T | T |

there is no row where all three sentences are simultaneously true!
jointly unsatisfiable

p 93 c1) ^{is}

$A \rightarrow A \therefore A$ valid?

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Really this is asking whether or not

$$A \rightarrow A \models A$$

| A | $A \rightarrow A$ | A |
|---|-------------------|---|
| T | T | T |
| F | T | F |

premise conclu

in this valuation,

$A \rightarrow A$ true but A is false
(premise) (conclusion)

Therefore,

$$A \rightarrow A \not\models A$$

So the argument

$A \rightarrow A \therefore A$ is not valid,

p.92 C2 | Ts

$$A \rightarrow (A \wedge \neg A) \therefore \neg A$$

valid?

Check if

$$A \rightarrow (A \wedge \neg A) \vDash \neg A$$

| A | $A \rightarrow (A \wedge \neg A)$ | $\neg A$ |
|---|-----------------------------------|----------|
| T | F | F |
| F | T | T |

premise conclusion

In this valuation premise is false, so it cannot be a counterexample

In this valuation, true premise true & conclu. ~~false~~

~~$A \rightarrow (A \wedge \neg A) \vDash \neg A$~~
NOT a counterexample

Therefore,

$$A \rightarrow (A \wedge \neg A) \vDash \neg A$$

Thus,

$$A \rightarrow (A \wedge \neg A) \therefore \neg A \text{ is valid}$$