

Equally, both sentences mean that if you do not catch a cold, then you must have worn a jacket. With this in mind, we might symbolize them as ' $(\neg D \rightarrow J)$ '.

Equally, both sentences mean that either you will wear a jacket or you will catch a cold. With this in mind, we might symbolize them as ' $(J \vee D)$ '.

All three are correct symbolizations. Indeed, in chapter 12 we will see that all three symbolizations are equivalent in TFL.

If a sentence can be paraphrased as 'Unless  $A$ ,  $B$ ,' then it can be symbolized as ' $(A \vee B)$ '.

Again, though, there is a little complication. 'Unless' can be symbolized as a conditional; but as we said above, people often use the conditional (on its own) when they mean to use the biconditional. Equally, 'unless' can be symbolized as a disjunction; but there are two kinds of disjunction (exclusive and inclusive). So it will not surprise you to discover that ordinary speakers of English often use 'unless' to mean something more like the biconditional, or like exclusive disjunction. Suppose someone says: 'I will go running unless it rains'. They probably mean something like 'I will go running iff it does not rain' (i.e., the biconditional), or 'either I will go running or it will rain, but not both' (i.e., exclusive disjunction). Again: be aware of this when interpreting what other people have said, but be precise in your writing.

## Practice exercises

A. Using the symbolization key given, symbolize each English sentence in TFL.

$M$ : Those creatures are men in suits.

$C$ : Those creatures are chimpanzees.

$G$ : Those creatures are gorillas.

1. Those creatures are not men in suits.  $\neg M$

2. Those creatures are men in suits, or they are not.  $M \vee \neg M$
3. Those creatures are either gorillas or chimpanzees.  $G \vee C$
4. Those creatures are neither gorillas nor chimpanzees.  $\neg(G \wedge C)$
5. If those creatures are chimpanzees, then they are neither gorillas nor men in suits.  $C \rightarrow \neg(G \wedge M)$
6. Unless those creatures are men in suits, they are either chimpanzees or they are gorillas.  $M \vee (C \vee G)$

**B.** Using the symbolization key given, symbolize each English sentence in TFL.

- A*: Mister Ace was murdered.  
*B*: The butler did it.  
*C*: The cook did it.  
*D*: The Duchess is lying.  
*E*: Mister Edge was murdered.  
*F*: The murder weapon was a frying pan.

1. Either Mister Ace or Mister Edge was murdered.
2. If Mister Ace was murdered, then the cook did it.
3. If Mister Edge was murdered, then the cook did not do it.
4. Either the butler did it, or the Duchess is lying.
5. The cook did it only if the Duchess is lying.
6. If the murder weapon was a frying pan, then the culprit must have been the cook.
7. If the murder weapon was not a frying pan, then the culprit was either the cook or the butler.
8. Mister Ace was murdered if and only if Mister Edge was not murdered.
9. The Duchess is lying, unless it was Mister Edge who was murdered.
10. If Mister Ace was murdered, he was done in with a frying pan.
11. Since the cook did it, the butler did not.
12. Of course the Duchess is lying!

meaning the same thing as  $\neg(Q \wedge R)$ , but as we saw in §5.2, this is very different from  $\neg(Q \wedge R)$ .

Strictly speaking, then,  $Q \wedge R$  is *not* a sentence. It is a mere *expression*.

When working with TFL, however, it will make our lives easier if we are sometimes a little less than strict. So, here are some convenient conventions.

First, we allow ourselves to omit the *outermost* brackets of a sentence. Thus we allow ourselves to write  $Q \wedge R$  instead of the sentence  $(Q \wedge R)$ . However, we must remember to put the brackets back in, when we want to embed the sentence into a more complicated sentence!

Second, it can be a bit painful to stare at long sentences with many nested pairs of brackets. To make things a bit easier on the eyes, we will allow ourselves to use square brackets, '[' and ']', instead of rounded ones. So there is no logical difference between  $(P \vee Q)$  and  $[P \vee Q]$ , for example.

Combining these two conventions, we can rewrite the unwieldy sentence

$$\neg \left( \left( (H \rightarrow I) \vee (I \rightarrow H) \right) \wedge (J \vee K) \right)$$

rather more clearly as follows:

$$\neg \left[ (H \rightarrow I) \vee (I \rightarrow H) \right] \wedge (J \vee K)$$

The scope of each connective is now much easier to pick out.

## Practice exercises

A. For each of the following: (a) Is it a sentence of TFL, strictly speaking? (b) Is it a sentence of TFL, allowing for our relaxed bracketing conventions?

1.  $(A)$  Yes -- the parentheses aren't necessary but are ok
2.  $J_{374} \vee \neg J_{374}$  Yes --  $J_{374}$  is a sentence letter.
3.  $\neg\neg\neg\neg F$  Yes -- because notnotnotF is a sentence bc notnotF is a sentence because notF is a sentence bc F is a sentence letter.

No -- although S is a sentence letter, "and S" is

- 4.  $\neg \wedge S$  not a sentence
- 5.  $(G \wedge \neg G)$  Yes
- 6.  $(A \rightarrow (A \wedge \neg F)) \vee (D \leftrightarrow E)$
- 7.  $[(Z \leftrightarrow S) \rightarrow W] \wedge [J \vee X]$
- 8.  $(F \leftrightarrow \neg D \rightarrow J) \vee (C \wedge D)$

#6: Yes. D and E are sentence letters, so  $D \leftrightarrow E$  is a sentence. F is a sentence so  $\neg F$  is a sentence. So A AND notF is a sentence. So  $A \rightarrow (A \wedge \neg F)$  is a sentence. Combining this one with  $D \leftrightarrow E$  gives #6.

#7 and #8 are similar

- B. Are there any sentences of TFL that contain no sentence letters? Explain your answer. No -- rules 2-6 require a sentence to exist to use. Rule 1 gives us a way to create a sentence from nothing. Rule 7 excludes any other expression from being a sentence. So since there is no alternative to rule 1 to create a sentence from nothing, we must have a sentence letter in every sentence of TFL.
- C. What is the scope of each connective in the sentence

$$[(H \leftrightarrow I) \vee (I \leftrightarrow H)] \wedge (J \vee K)$$