

∴ Abe Lincoln was from France.

1. If the world ends today, then I will not need to get up tomorrow morning.
 2. I will need to get up tomorrow morning.
- ∴ The world will not end today.

1. Joe is now 19 years old.
 2. Joe is now 87 years old.
- ∴ Bob is now 20 years old.

Question: "What is an example of a premise that is always false?" Answer: "A equals 6 and A equals 7"

B. Could there be:

1. A valid argument that has one false premise and one true premise? #1: YES - in this scenario, a counterexample (situation where the premises are all true and conclusion false) can't exist since one premise is false
2. A valid argument that has only false premises? #2: YES - again we cannot construct a counterexample if even one premise is false, let alone all of them
3. A valid argument with only false premises and a false conclusion? #3: YES - same as above -- cannot construct a counterexample
4. An invalid argument that can be made valid by the addition of a new premise? #4: YES -- just add an always-false premise
5. A valid argument that can be made invalid by the addition of a new premise? #5: NO -- suppose we add an always-false premise. Then by above the resulting argument is valid. So suppose we add an always-true premise. If we had a counterexample here (making the new argument invalid) then consider that counterexample applied to the original argument. It should still be a counterexample, but we know the original argument is valid so no counterexample exists.

In each case: if so, give an example; if not, explain why not.

Practice exercises

A. For each of the following: Is it a necessary truth, a necessary falsehood, or contingent?

1. Caesar crossed the Rubicon. #1: contingent -- to see true: imagine Julius Cesar crossing the river Rubicon in ancient Rome; to see false: imagine a person named Cesar crossing a street
2. Someone once crossed the Rubicon. #2: contingent -- Rubicon could be the river (in which case Julius C. crossed it) or it could be a thing in space no one has visited
3. No one has ever crossed the Rubicon. #3: contingent -- same reasoning as before
4. If Caesar crossed the Rubicon, then someone has. #4: necessarily true (only if Cesar is a person and "someone" only refers to people, not, e.g. cats)
5. Even though Caesar crossed the Rubicon, no one has ever crossed the Rubicon. #5: necessarily false (but could be contingent if "Rubicon" referred to different things in both instances)
6. If anyone has ever crossed the Rubicon, it was Caesar. #6: contingent -- imagine Julius Cesar crossing river Rubicon (false -- other people did too) or a one-person spaceship with an astronaut who is only person who crossed a river on some distant planet

B. For each of the following: Is it a necessary truth, a necessary falsehood, or contingent?

1. Elephants dissolve in water.
2. Wood is a light, durable substance useful for building things.
3. If wood were a good building material, it would be useful for building things.
4. I live in a three story building that is two stories tall.
5. If gerbils were mammals they would nurse their young.

C. Which of the following pairs of sentences are necessarily equivalent?

1. Elephants dissolve in water. #1: yes -- assuming disintegration is the same as dissolution (maybe chemists will disagree)
If you put an elephant in water, it will disintegrate.
2. All mammals dissolve in water. #2 no -- imagine if elephants were the only mammal that dissolved in water
If you put an elephant in water, it will disintegrate.
3. George Bush was the 43rd president.
Barack Obama is the 44th president. #3: not equivalent -- these have nothing to do with each other
4. Barack Obama is the 44th president.
Barack Obama was president immediately after the 43rd president. #4: yes equivalent -- because 44 is the number immediately after 43
5. Elephants dissolve in water.
All mammals dissolve in water.

D. Which of the following pairs of sentences are necessarily equivalent?

1. Thelonious Monk played piano.
John Coltrane played tenor sax.
2. Thelonious Monk played gigs with John Coltrane.
John Coltrane played gigs with Thelonious Monk.
3. All professional piano players have big hands.
Piano player Bud Powell had big hands.
4. Bud Powell suffered from severe mental illness.
All piano players suffer from severe mental illness.
5. John Coltrane was deeply religious.
John Coltrane viewed music as an expression of spirituality.

E. Consider the following sentences:

- G₁ There are at least four giraffes at the wild animal park.
- G₂ There are exactly seven gorillas at the wild animal park.
- G₃ There are not more than two Martians at the wild animal park.
- G₄ Every giraffe at the wild animal park is a Martian.

Now consider each of the following collections of sentences. Which are jointly possible? Which are jointly impossible?

1. Sentences G₂, G₃, and G₄ #1: jointly poss -- 7 gorillas & 1 martian & 0 giraffes -- then all true
2. Sentences G₁, G₃, and G₄ #2: jointly imposs -- all giraffes are martians, there's >=4 giraffes but there should not be more than 2 martians
3. Sentences G₁, G₂, and G₄ #3: jointly possible -- 7 gorillas, 4 giraffes, 5 martians (4 of which happen to be giraffes)
4. Sentences G₁, G₂, and G₃ #4: jointly possible -- 7 gorillas, 4 giraffes, 2 martians

F. Consider the following sentences.

- M₁ All people are mortal.
- M₂ Socrates is a person.
- M₃ Socrates will never die.

M_4 Socrates is mortal.

Which combinations of sentences are jointly possible? Mark each “possible” or “impossible.”

1. Sentences M_1 , M_2 , and M_3
2. Sentences M_2 , M_3 , and M_4
3. Sentences M_2 and M_3
4. Sentences M_1 and M_4
5. Sentences M_1 , M_2 , M_3 , and M_4

G. Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument that has one false premise and one true premise
2. A valid argument that has a false conclusion
3. A valid argument, the conclusion of which is a necessary falsehood
4. An invalid argument, the conclusion of which is a necessary truth
5. A necessary truth that is contingent
6. Two necessarily equivalent sentences, both of which are necessary truths
7. Two necessarily equivalent sentences, one of which is a necessary truth and one of which is contingent
8. Two necessarily equivalent sentences that together are jointly impossible
9. A jointly possible collection of sentences that contains a necessary falsehood
10. A jointly impossible set of sentences that contains a necessary truth

H. Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument, whose premises are all necessary truths, and whose conclusion is contingent

#2: possible -- if an always false premise existed, then no counterexample can be constructed

#1: possible -- a false premise never allows a counterexample (true premises, false conclusion) to exist