Written HW7 – MATH 2502 Spring 2021

Due by 24 February for timely completion credit

Consider the so-called "lower incomplete gamma function"

$$\gamma_s(x) = \int_0^x t^{s-1}e^{-t}\mathrm{d}t. \tag{1}$$

is a function of two variables "s" and "x".

This function has published applications in numerous areas of science such as protein folding in biology, "Gaussian orbitals" in quantum chemistry, and ecological systems theory (see `https://dlmf.nist.gov/8.24`). It is related to the so-called "gamma function" Γ which is one of the most important functions in mathematics (see `https://en.wikipedia.org/wiki/Gamma_function`).

#1. Write the integral corresponding to $\gamma_1(x)$ by setting $s = 1$ in (1).

#2. Solve the integral you found in #1.

#3. Find $\gamma_2(x)$ using integration by parts. Your answer will be a function of the variable x.

#4. Use Desmos to plot the functions $\gamma_1(x)$, $\gamma_2(x)$, $\gamma_3(x)$, $\gamma_4(x)$, and $\gamma_5(x)$ on the same plot. You can use the integral formulation to accomplish this. (*note: you were asked to do something similar in WHW6 #4*).

#5. Perform integration by parts on the integral (1) by letting $u = t^{s-1}$ and $\mathrm{d}v = e^{-t}$. Be careful not to use "x" as a variable except as a bound of the integral or being plugged in for t when appropriate!!

#6. Note that in #5, the "$s - 1$" you should have gotten that appears in the integral is not dependent on t – it is a constant with respect to t (no different than something like "7") – so pull it out of the integral like you would any other constant.

#7. Now you should have

$$\gamma_s(x) = \text{"a function of } x\text{"} + (s-1)\text{"some kind of integral"}. \tag{2}$$

Which lower incomplete gamma function does the integral in that expression correspond to (*in other words, what should "s" be replaced by in* (1) *to obtain what you've got here?*)? Rewrite the expression (2) by filling in the "some kind of integral" expressed as the correct lower incomplete gamma function and filling in the "function of x" part you discovered in #5. This expression is known as the "recurrence relation" associated to the lower incomplete gamma function and is fundamental to its use.