Written HW6 – MATH 2502 Spring 2021

Due by 19 February for timely completion credit

Consider the curve $f(t) = \dfrac{1}{t}$ on the interval $[1, x]$, where $x > 1$ is some number (*note: you don't pick x – use "x" in your calculations*).

#1. Sketch the curve and its shadow region.

#2. Find the volume of the solid obtained by taking the region defined on top by $f(t)$ and on the bottom by the horizontal axis and rotating it about the horizontal axis. Use your preferred method to find the volume (*note: the volume will contain the variable x*). All details must be shown when computing the integral.

#3. **Set up but do not evaluate** the integral that computes the surface area obtained when rotating the curve $f(t)$ over $[1, x]$ about the horizontal axis. (*note: the surface area will contain the variable x*).

#4. Use Desmos to plot the integral found in #3 above as a function of x (*note: I've done this kind of thing numerous times in class – see e.g., 4 minutes and 30 seconds into the lecture video for 15 February*). On the same plot, also plot the function $2\pi \ln(x)$ — which is bigger when $x > 1$? Include your plots in your answer.

#5. Compute the limit of the volume computed in #1 as $x \to \infty$ (this resembles a calculus 1 problem). Use your plots in #4 to conclude what the surface area computed in #2 becomes as $x \to \infty$.

#6. "Gabriel's horn" is the surface of revolution formed by rotating the curve $\dfrac{1}{x}$ lying above the infinite interval $[0, \infty)$ across the x-axis. Based on your answer to #5, what conclusion can you make about the volume and surface area of Gabriel's horn? It is often said that "Gabriel's horn can be filled with paint, but it can never be painted" – explain what that means in the context of your calculations.