Written HW20 – MATH 2502 Spring 2021

**Due by 23 April for timely completion credit**

1. Start with the geometric series $\dfrac{1}{1-r} = \displaystyle\sum_{k=0}^{\infty} r^k$, replace $r$ with $-t^2$, and then integrate to find a power series for the arctan function:

$$\arctan(x) = \int_0^x \frac{1}{1+t^2}\mathrm{d}t.$$

2. Start with the power series $e^t = \displaystyle\sum_{k=0}^{\infty} \frac{t^k}{k!}$, replace $t$ with $-t^2$, and then integrate to find a power series for the error function erf:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\mathrm{d}t.$$

3. Start with the power series $\sin(t) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!}$, replace $t$ with $t^2$, and then integrate to find a power series for the Fresnel $S$ function

$$S(x) = \int_0^x \sin(t^2)\mathrm{d}t.$$

4. As was done in the 19 April class, provide a plot for each of the answers (separate plot for each) containing the "actual function" as well as the series approximation to it up to four terms.