Some series for some functions

Ex: What is $\int_0^x \frac{1}{1+t^4}\,dt$?

We will find power series.

$(a^b)^c = a^{bc}$

Start with

$$\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k \sim |r|<1 \quad -1<r<1$$

Let $r = -t^4$:

$$\frac{1}{1-(-t^4)} = \sum_{k=0}^{\infty} (-1)^k (t^4)^k$$

$$\frac{1}{1+t^4} = \sum_{k=0}^{\infty} (-1)^k t^{4k}$$

$$\downarrow \int_0^x \_\_\, dt$$

$$\int_0^x \frac{1}{1+t^4}\,dt = \int_0^x \sum_{k=0}^{\infty} (-1)^k t^{4k}\,dt$$

$$= \sum_{k=0}^{\infty} (-1)^k \int_0^x t^{4k}\,dt$$

$$= \sum_{k=0}^{\infty} (-1)^k \left[\frac{t^{4k+1}}{4k+1} \right]_{t=0}^{t=x}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+1}}{4k+1}$$

Ex: Find series for $f(x) = \int_0^x t e^{-t^4}\,dt$

Start from $e^t = \sum_{k=0}^{\infty} \frac{t^k}{k!}$ $\qquad t \cdot t^k = t^{k+1}$

$$e^{-t^4} = \sum_{k=0}^{\infty} \frac{(-1)^k t^{4k}}{k!}$$

$$te^{-t^4} = \sum_{k=0}^{\infty} \frac{(-1)^k t^{4k+1}}{k!}$$

$\downarrow \int_0^x \ldots dt$

$$\int_0^x te^{-t^4}\,dt = \int_0^x \sum_{k=0}^{\infty} \frac{(-1)^k t^{4k+1}}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int_0^x t^{4k+1}\,dt \qquad \frac{t^{4k+2}}{4k+2}\Big|_0^x$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{x^{4k+2}}{4k+2} = \frac{x^2}{2} - \frac{x^6}{6} + \frac{x^{10}}{2 \cdot 10} - \frac{x^{14}}{3! \cdot 14} + \ldots$$

Ex: Fresnel C

$$C(x)=\int_0^x \cos(t^2)\,dt$$

$\sin(-x)=-\sin(x)$

$\cos(-x)=\cos(x)$

Soln: series for cosine:

$(t^2)^{2k}=t^{4k}$

$$\cos(t)=\sum_{k=0}^{\infty}\frac{(-1)^k t^{2k}}{(2k)!}$$

$$\cos(t^2)=\sum_{k=0}^{\infty}\frac{(-1)^k t^{4k}}{(2k)!}$$

$\downarrow \int_0^x \ldots dt$

$$\int_0^x \cos(t^2)\,dt=\int_0^x \sum_{k=0}^{\infty}\frac{(-1)^k t^{4k}}{(2k)!}\,dt$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\int_0^x t^{4k}\,dt \qquad \left.\frac{t^{4k+1}}{4k+1}\right|_0^x$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\,\frac{x^{4k+1}}{4k+1}$$

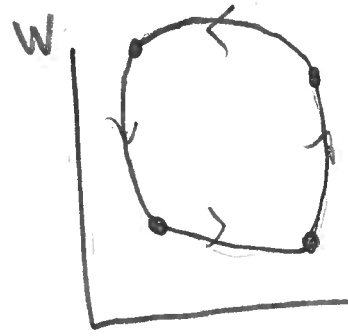
Matrix special functions

Matrix: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

JPG
GIF

Linear Algebra

wolves eat sheep



$$\begin{cases} 5 = x - 2y \\ 3 = x + y \end{cases}$$

$$\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \sim \end{pmatrix}$$

$$\sin\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right)$$

$$e^{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \sum_{k=0}^{\infty} \frac{\begin{pmatrix} a & b \\ c & d \end{pmatrix}^k}{k!}$$

Calc 3: calculus in higher dimension