Taylor series centered at c:

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

Continue from last time:

$$f(x) = \frac{1}{3-x} = \frac{1}{2}\sum_{k=0}^{\infty} \frac{1}{2^k}(x-1)^k$$

In order to have used geometric series formula with $r = \frac{x-1}{2}$, we need to enforce $|r|<1$:

$|x|<1$

$-1<x<1$

$|-0.25| = 0.25$

$$\left|\frac{x-1}{2}\right| < 1$$

$$-1 < \frac{x-1}{2} < 1$$

$\downarrow$ mult by 2

$$-2 < x-1 < 2$$

$\downarrow$ add 1

$$-1 < x < 3 \sim \text{IoC: } (-1,3)$$

Ex: Find Taylor series for $g(x) = \frac{5}{2x-3}$ centered at $c=-3$.

power series centered at $c=-3$ has variable terms that look like $(x-(-3))^k = (x+3)^k$

$$\frac{1}{1-r} = \sum_0^\infty r^k$$

$$g(x) = \frac{5}{2x-3}$$

$$= \frac{5}{2(x+3\underbrace{-3)}_{=0}-3} = \frac{5}{2(x+3)-6-3}$$

$$=$$

$$= \frac{5}{2(x+3)-9}$$

$$= \frac{5}{(2(x+3)-9)} \frac{(-1)}{(-1)}$$

$$= \frac{-5}{9-2(x+3)} \left(\frac{1/9}{1/9}\right)$$

$$= \frac{-5/9}{1-\frac{2}{9}(x+3)} = -\frac{5}{9} \frac{1}{1-\frac{2}{9}(x+3)}$$

$+\frac{9}{2} - \frac{6}{2}$

$r = \frac{2}{3}(x+3)$

$$= -\frac{5}{9} \sum_{k=0}^{\infty} \frac{2^k}{9^k}(x+3)^k$$

as long as

$$\left|\frac{2}{9}(x+3)\right| < 1$$

$$-1 < \frac{2}{9}(x+3) < 1$$

$$-\frac{9}{2} < x+3 < \frac{9}{2}$$

$$-\frac{15}{2} < x < \frac{3}{2}$$

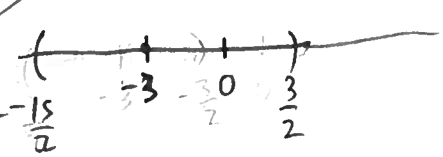
$-3 - \frac{3}{2}$

$-\frac{6}{2} - \frac{3}{2}$

$\frac{3}{2} - 3$

$= \frac{3}{2} - \frac{6}{2}$

IoC: $\left(-\frac{15}{2}, \frac{3}{2}\right)$



Same instructions : $h(x) = \frac{7}{5x-1}$ centered at $c = 4$

$(x-c)^k$

$(x-4)^k$

$\frac{1}{1-r} = \sum_0^{\infty} r^k$

$$h(x) = \frac{7}{5x-1} = \frac{7}{5(x-4+4)-1}$$

no minus → $= \frac{7}{5(x-4)+19}$

force minus → $= \frac{7}{-[-5(x-4)]+19}$

$y = \frac{20}{5}$

$$= \frac{7}{19-[-5(x-4)]}\left(\frac{1/19}{1/19}\right)$$

$$= \frac{7}{19} \cdot \frac{1}{1-\left[\frac{-5}{19}(x-4)\right]}$$

geoseries w/ $r = -\frac{5}{19}(x-4)$ → $= \frac{7}{19}\sum_{k=0}^{\infty} \frac{(-1)^k 5^k}{19^k}(x-4)^k$

valid for

$\left|-\frac{5}{19}(x-4)\right| < 1$

$-1 < -\frac{5}{19}(x-4) < 1$

$1 > \frac{5}{19}(x-4) > -1$

$-1 < \frac{5}{19}(x-4) < 1$

$-\frac{19}{5} < x-4 < \frac{19}{5}$

$\frac{1}{5} < x < \frac{39}{5}$

IoC : $\left(\frac{1}{5}, \frac{39}{5}\right)$

Differentiation + Integration of Power Series

How do I compute

$$\frac{d}{dx}\left(\sum_{k=0}^{\infty} a_k x^k\right) = \frac{d}{dx}\left(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots\right)$$

$$= a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots$$

$$f(x) = \frac{1}{(1-x)} \rightarrow \boxed{f'(x) = \frac{1}{(1-x)^2}}$$

But also

$$f(x) = \sum_{k=0}^{\infty} x^k, \quad |x|<1$$

$$\boxed{f'(x) = \sum_{k=0}^{\infty} kx^{k-1} = \sum_{k=1}^{\infty} kx^{k-1}}$$

k=0 term is zero!

Notice that we now have, for $|x|<1$:

$$\frac{1}{(1-x)^2} = f'(x) = \sum_{k=1}^{\infty} kx^{k-1}$$

How do I compute

$$\int_0^t \sum_{k=0}^{\infty} a_k x^k \, dx = \sum_{k=0}^{\infty} a_k \int_0^t x^k \, dx$$

$$= \sum_{k=0}^{\infty} a_k \left[\frac{x^{k+1}}{k+1}\right]_0^t$$

$$= \sum_{k=0}^{\infty} \frac{a_k t^{k+1}}{k+1}$$

Find series for $\ln(1+x)$.

$$f(x) = \frac{1}{1-x} \longrightarrow \boxed{\int f(x)\,dx = -\ln(1-x)}$$

$u = 1-x$

$-du = dx$

$$f(x) = \sum_{k=0}^{\infty} x^k, \quad |x| < 1$$

$$\boxed{\int f(x)\,dx = \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1}}$$

$$\ln(1-x) = \int f(x)\,dx = \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1}$$

$$= \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$$