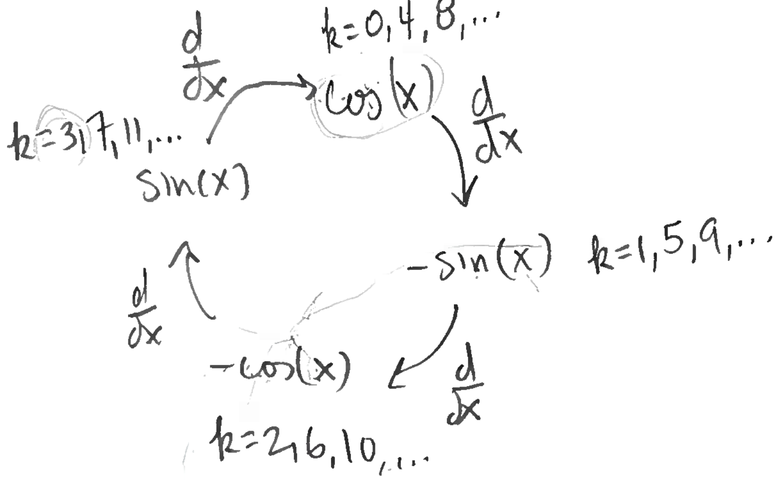
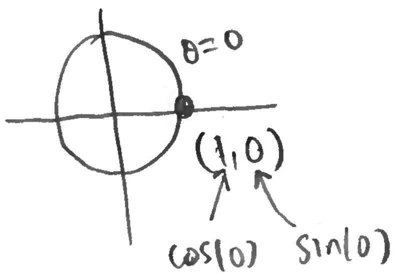
Ex: Taylor series of cos(x) centered at c=0



k	$\cos^{(k)}(0)$
0	1
1	0
2	-1
3	0
4	1
5	0
6	-1
7	0
8	1

repeats



k=0 k=1 k=2

$$\Rightarrow \cos(x) = \frac{1(x-0)^0}{\underbrace{0!}_{=1}} + 0 - \frac{1x^2}{2!} + 0 + \frac{x^4}{4!} + 0 - \frac{x^6}{6!} + 0 + \frac{x^8}{8!} + \dots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots$$

We know:

a while ago $\longrightarrow$ $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \ldots$

yesterday $\longrightarrow$ $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

prev page $\longrightarrow$ $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

k	i^k
0	1
1	i
2	-1
3	$-i$
4	1
5	i
6	-1
7	$-i$
⋮	

Recall: imaginary number $i = \sqrt{-1} \quad \Longleftrightarrow \quad i^2 = -1$

i^{57}

$$\begin{tikzcd} & i \arrow[dr, "\text{mult by } i"] & \\ 1 \arrow[ur, "\text{mult by } i"] & & -1 \arrow[dl, "\text{mult by } i"] \\ & -i \arrow[ul, "\text{mult by } i"] & \end{tikzcd}$$

Weird question: What is e^{ix}?

By power series:

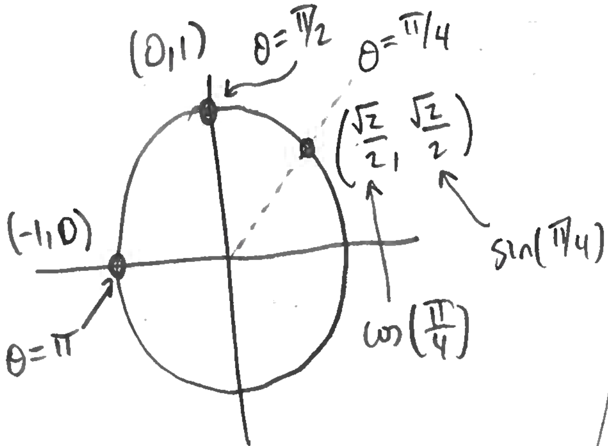
$$e^{ix} = 1 + \frac{(ix)}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!}$$

$$= 1 + ix - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} + i\frac{x^5}{5!} - \frac{x^6}{6!} - i\frac{x^7}{7!} + \ldots$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\right)$$

$$= \cos(x) + i\sin(x)$$

Euler's Formula: $\boxed{e^{ix} = \cos(x) + i\sin(x)}$



$$e^{i\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

$$e^{i\pi} = -1 + i(0)$$

$$\boxed{e^{i\pi} = -1}$$

$$\boxed{e^{i\pi} + 1 = 0}$$

$$e^{i\frac{\pi}{2}} = 0 + i$$

$$e^{i\pi/2} = i$$