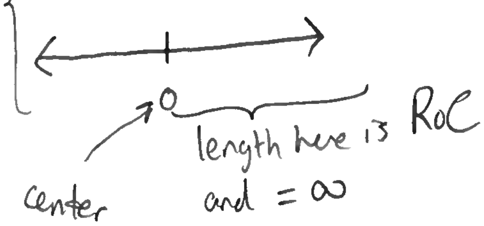
Let $\sum a_k (x-c)^k$ be a power series.

Def: The <u>interval of convergence</u> (IoC) (Yesterday I wrote "región") is the set of x-values for which power series converges.

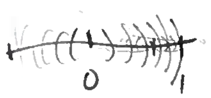
The <u>radius of convergence</u> (RoC) is the distance from the center of the series to its edge.

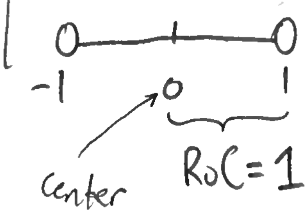
3 examples from last time

① $e \approx 2.71\ldots$ $\left(e^x = \right) \boxed{\sum_{k=0}^{\infty} \frac{x^k}{k!}}$ ~ IoC: $-\infty < x < \infty$ or

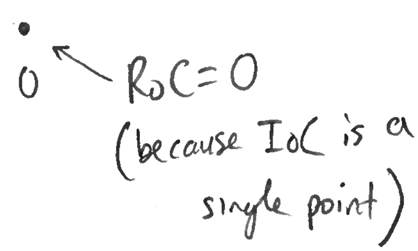


② $\left(\frac{1}{1-x} = \right) \sum_{k=0}^{\infty} x^k$ ~ IoC: $-1 < x < 1$ or

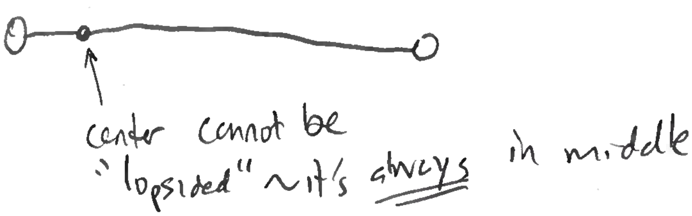




③ $\sum_{k=0}^{\infty} x^k \cdot k!$ ~ IoC: $x = \{0\}$ ← center



Not happen:



Find IoC and RoC.... (all centered at 0)

Ex: $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k+1}$

Soln: Use ratio test ~ $a_k = \dfrac{(-1)^k x^k}{k+1}$

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \frac{|x|^{k+1}}{k+2}\cdot\frac{k+1}{\cancel{|x|^k}}$$

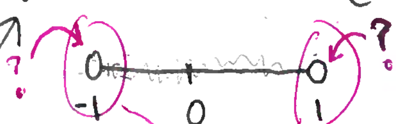
$$= |x| \boxed{\lim_{k\to\infty}\frac{k+1}{k+2}} \to 1$$

$$= \boxed{|x| < 1}$$

enforce in order to get convergence

$\Rightarrow -1 < x < 1$ i.e. IoC ~ $(-1,1)$

these x's work!



ratio test does not tell us what happens at endpoints (b/c test is inconclusive if $|x|=1$)

But what happens at endpoints?

We have to check $x=1$, $x=-1$ seperately!!

$\boxed{x=1}$ ✓

Series becomes

$$\sum_{k=0}^{\infty}\frac{(-1)^k 1^k}{k+1} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}$$

alternating series

Alt. series test: $b_k = \dfrac{1}{k+1}$

$\lim_{k\to\infty} b_k = 0 \Rightarrow$ series ✓ converges

negatives cancel out

⊗ $x=-1$ Series is $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k(-1)^k}{k+1}$

$= \displaystyle\sum_{k=0}^{\infty}\frac{1}{k+1}$, which diverges because if $b_k = \dfrac{1}{k}$

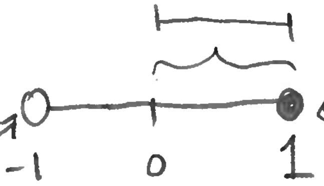
★ $\displaystyle\lim_{k\to\infty}\frac{a_k}{b_k} = \lim_{k\to\infty}\frac{k}{k+1} = 1$

★ $\sum b_k = \sum \frac{1}{k} = \infty$ (harmonic)

$\Downarrow$

$\displaystyle\sum_{k=0}^{\infty}\frac{1}{k+1}$ diverges

IoC:



shaded in means $x = 1$ is a place where series converges

not shaded in $\Rightarrow$ series diverges at $x = -1$

RoC: 1 ~ distance from center to edge

Ex: $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{5^k}$ ← centered at 0

Everything has power k... root test!

$$a_k = \frac{(-1)^k x^k}{5^k}$$

$$\lim_{k\to\infty} \sqrt[k]{\left|\frac{(-1)^k x^k}{5^k}\right|} = \lim_{k\to\infty} \sqrt[k]{\frac{|x|^k}{5^k}}$$

$$= \lim_{k\to\infty} \frac{|x|}{5} = \frac{|x|}{5}$$

< 1 ⇒ conv abs

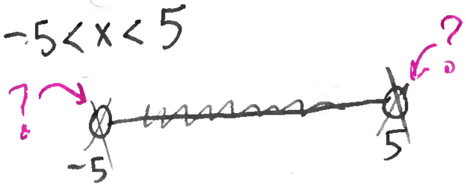
= 1 ⇒ inconclusive

> 1 ⇒ diverge

$$\frac{|x|}{5} < 1 \Rightarrow |x| < 5$$

$$\Rightarrow -5 < x < 5$$

$$(-5)^k = (-1)^k (5)^k$$



$x = -5$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \cancel{(-5)^k}}{\cancel{5^k}} = \sum_{k=0}^{\infty} 1$$

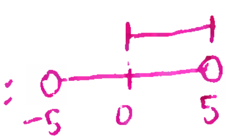
diverges (test for div)

$x = 5$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \cancel{5^k}}{\cancel{5^k}} = \sum_{k=0}^{\infty} (-1)^k$$

diverges (test for div)

IoC:



$(-5, 5)$ or $-5 < x < 5$

RoC: 5

Ex: $\displaystyle\sum_{k=0}^{\infty} \frac{x^{5k}}{k!}$

Ratio test: $a_k = \dfrac{x^{5k}}{k!}$

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \frac{|x|^{\cancel{5k}+5}}{(k+1)!}\,\frac{k!}{\cancel{|x|^{5k}}}$$

$$= \lim_{k\to\infty} \frac{|x|^5}{k+1} = 0 < 1$$

always < 1 for ALL x

IoC: $\mathbb{R}$ $\quad (-\infty, \infty)$

$0 \quad\quad -\infty < x < \infty$

RoC: ∞