$k=0$

$0+?=4$

Reindex:

went up by 3

replaced k with k+3

$$\sum_{k=3}^{\infty} \frac{x^{k+1}}{k-1} = \sum_{k=0}^{\infty} \frac{x^{k+4}}{k+2}$$

went up by 3

start value of k went down by 3

$$= \frac{x^4}{2} + \frac{x^5}{3} + \frac{x^6}{4} + \dots$$

replace k with k+4

up by 4

$$\sum_{k=6}^{\infty} \frac{(k+1)x^k}{k-3} = \sum_{k=2}^{\infty} \frac{(k+5)x^{k+4}}{k+1}$$

down by 4

$$\frac{7x^6}{3} + \frac{8x^7}{4} + \frac{9x^8}{5} + \dots$$

replace k w/ k-6

down by 6

$$\sum_{k=1}^{\infty} \frac{(k-10)e^k}{k!}x^k = \sum_{k=7}^{\infty} \frac{(k-16)e^{k-6}}{(k-6)!}x^{k-6}$$

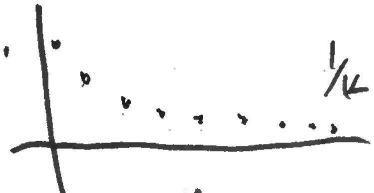
up by 6

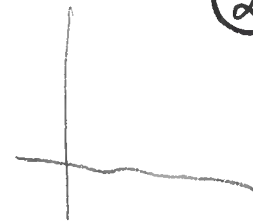
$$\frac{-9e}{1!}x + \frac{(-8)e^2}{2!}x^2 + \frac{(-7)e^3}{3!}x^3 + \dots$$

Harmonic series

$$\sum_{K=1}^{\infty} \frac{1}{K} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

to ∞





diverges even though

$$\lim_{K \to \infty} \frac{1}{K} = 0$$

$$\int_0^1 \frac{\ln(x)}{\sqrt{x}}$$

vs

$$\int_0^1 \frac{\ln(x)}{x}$$

diverges VERY slowly

FACT: Sometimes when $a_k \to 0$, $\sum^{\infty} a_k$ exists

but other times when $a_k \to 0$, $\sum^{\infty} a_k$ Does not exist!

Geometric series

$$\sum_{k=0}^{\infty} ar^k = a\underbrace{r^0}_{=1} + ar^1 + ar^2 + ar^3 + \dots$$

FROM 12 May notes!!

ex) $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 2$

ex) $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots = 3/2$

"magic"

<u>Book</u>: if $|r|<1$, then geometric series converges

and

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$$

fails the condition

if $r=1/2$, $a=1$:

$$\sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^k = \frac{1}{1-1/2} = \frac{1}{1/2} = 2$$

if $r=1/3$, $a=1$:

$$\sum_{k=0}^{\infty} \left(\tfrac{1}{3}\right)^k = \frac{1}{1-1/3} = \frac{1}{2/3} = 3/2$$

if $r=1$, $a=1$:

$$\sum_{k=0}^{\infty} 1^k = 1+1+1+1+1+\dots = \infty$$

(can't apply formula b/c here $|r|=1$)

if $r=2$, $a=1$:

$$\sum_{k=0}^{\infty} 2^k = 1+2+4+8+\dots = \infty$$

Series diverges

IF it had a value (it DOESN'T) maybe assigning its value to be

$$\frac{1}{1-2} = -1$$ makes sense?

Grandi's series

DOES NOT Converge

$r = -1$, $a = 1$

$$\sum_{k=0}^{\infty} (-1)^k = 1 - 1 + 1 - 1 + 1 \ldots$$

$|r| = 1$

so geometric series does not converge

BUT if it had to have a value, maybe use

$$\frac{1}{1-(-1)} = \frac{1}{2} \text{ makes sense?}$$

This pink ink technique is called "analytic continuation".

Look into "divergent series" assigning values to