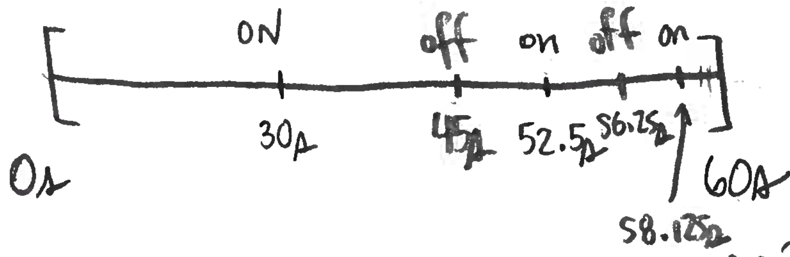
Thompson's lamp

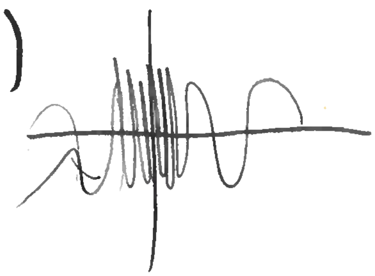
on-off switch for a light ~

experiment: flip switch every time you arrive at exactly half of remaining time



Q: At 60s ~ is light on or off?

Reminds me of graph of $\sin\left(\frac{1}{x}\right)$



(side: Warsaw circle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associtative law
$a+(b+c)=(a+b)+c$

Grandi's series

$S = 1 - 1 + 1 - 1 + 1 - 1 + \ldots\ldots$

Book
this series is divergent

What's its value? Call is S for now.

$S = (1-1)+(1-1)+(1-1)+\ldots = 0+0+0+\ldots \boxed{=0}$

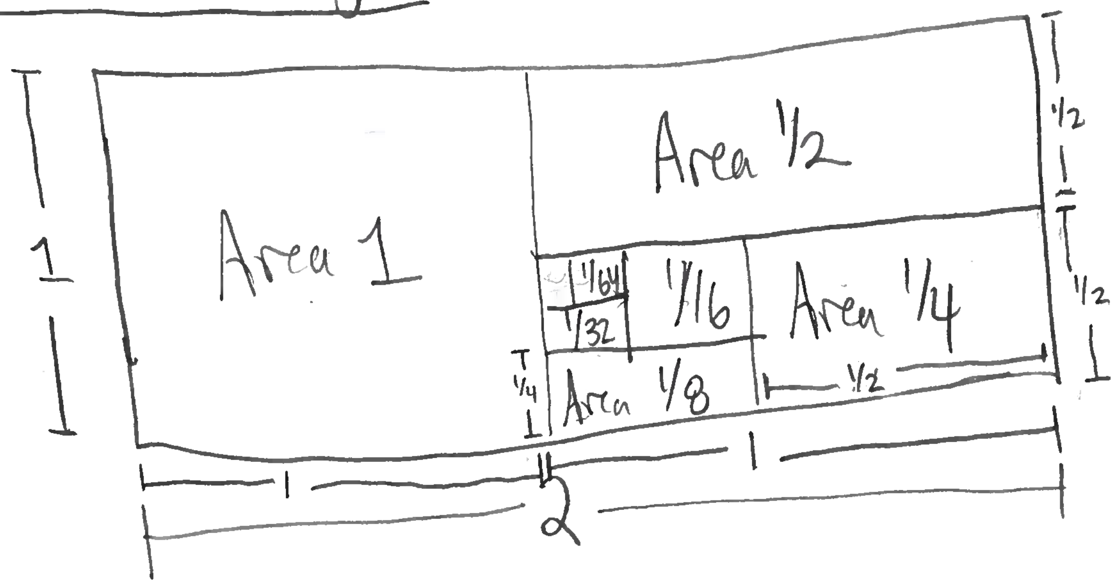
$S = 1+(-1+1)+(-1+1)+\ldots = 1+0+0+0+\ldots \boxed{=1}$

$-S = -1+1-1+1-1+1-1+\ldots$

$1-S = 1-1+1-1+1-1+\ldots = S$

$\rightarrow 1-S = S \rightarrow 1 = 2S \rightarrow \boxed{S = \frac{1}{2}}$

Consider rectangle



Total Area = 2

Split 1: $2 = 1 + 1$

Split 2: $2 = 1 + \frac{1}{2} + \frac{1}{2}$

Split 3: $2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

Split 4: $2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$

⋮

Split N: $2 = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n}$

⋮

Conclusion:

$$2 = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \dots$$

Claim: $\ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \dots$

$$\ln(2) = 1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \frac{1}{5} - \frac{1}{10} - \frac{1}{12} + \frac{1}{7} - \dots$$

$$2\cancel{\ln}(2) = \left(2 - \frac{2}{2}\right) - \frac{2}{4} + \left(\frac{2}{3} - \frac{2}{6}\right) - \frac{2}{8} + \left(\frac{2}{5} - \frac{2}{10}\right) - \frac{2}{12} + \dots$$

$$= (2-1) - \frac{1}{2} + \left(\frac{2}{3} - \frac{1}{3}\right) - \frac{1}{4} + \left(\frac{2}{5} - \frac{1}{5}\right) - \frac{1}{6} + \dots$$

$$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

$$= \ln(2)$$ **ABSURD**

In fact,

$$\ln(2) \neq 1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \dots$$

Actually:

$$1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \frac{1}{5} - \frac{1}{10} - \frac{1}{12} + \frac{1}{7} - \dots = \ln(2) + \frac{1}{2}\ln\left(\frac{1}{2}\right)$$

$$2\left(\ln(2) + \frac{1}{2}\ln\left(\frac{1}{2}\right)\right) = 2\ln(2) + \ln(1/2)$$
$$= \ln(4) + \ln(1/2)$$
$$= \ln\left(4 \cdot \frac{1}{2}\right) = \ln(2)$$

(TURNS OUT: series for $\ln(2)$ can be rearranged to get **ANY** real # you want)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

$$= 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}$$

Basel problem (solved by Euler)

$$1 + \frac{1}{2^{\text{even}}} + \frac{1}{3^{\text{even}}} + \dots = \frac{\pi^{\#_1}}{\#_2}$$

Apéry's constant

$$1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \dots = ??$$

(no "closed form formula")

shown in 1970's this number is irrational

One more

a sense in which

$$1 + 2 + 3 + 4 + 5 + 6 + \dots = -\frac{1}{12}$$

weird