If limit of a sequence exists, then we say the sequence <u>converges</u>. Otherwise we say the sequence <u>diverges</u>.

<u>Theorem 5.1</u>: Suppose $a_n = f(n)$. If

$$\lim_{x\to\infty} f(x) = L, \text{ then}$$

$$\lim_{n\to\infty} a_n = L.$$

<u>Ex</u>: $a_n = 1 - \frac{1}{n}$

Consider $f(x) = 1 - \frac{1}{x}$, then $a_n = f(n)$.

Since $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} 1 - \cancelto{0}{\frac{1}{x}} = 1$

we can conclude

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1 - \frac{1}{n} = 1$$

<u>Limit Laws</u> Let $\{a_n\}, \{b_n\}$ be given seqs, let $c \in \mathbb{R}$ (is a real number), and assume $\lim_{n\to\infty} a_n = A$ and $\lim_{n\to\infty} b_n = B$.

i) $\lim_{n\to\infty} c = c$

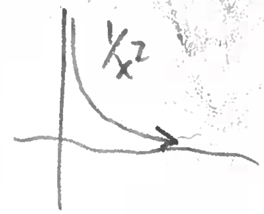
ii) $\lim_{n\to\infty} c a_n = cA$

iii) $\lim_{n\to\infty} a_n \pm b_n = A \pm B$

iv) $\lim_{n\to\infty} a_n \cdot b_n = AB$

v) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$ (provided $B \neq 0$ & $b_n \neq 0$ for all sufficiently large n)

Ex: $\lim_{n\to\infty} 5-\frac{3}{n^2} = \lim_{n\to\infty} 5 - 3\lim_{n\to\infty}\frac{1}{n^2}$



$= 5-3(0)$

$=5$

$$\lim_{n\to\infty}\frac{\overset{\infty}{3n^4-7n^2+5}}{6-4n^4} = -\frac{3}{4}$$

mult by $1 = \frac{1/n^4}{1/n^4}$ → ALGebra

L'Hôpital

$$= \lim_{n\to\infty}\frac{3-\overset{0}{7/n^2}+\overset{0}{5/n^4}}{\underset{0}{6/n^4}-4}$$

$= -3/4$

Thm 5.1 L.H. $\frac{\infty}{\infty}$

$$\overset{L.H.}{=} \lim_{x\to\infty}\frac{\frac{d}{dx}\left[3x^4-7x^2+5\right]}{\frac{d}{dx}\left[6-4x^4\right]}$$

$$= \lim_{x\to\infty}\frac{12x^3-14x}{-16x^3} \to \frac{\infty}{-\infty}$$

16
3
48

$$\overset{L.H.}{=} \lim_{x\to\infty}\frac{36x^2-14}{-48x^2} \to \frac{\infty}{-\infty}$$

48

$$\overset{L.H.}{=} \lim_{x\to\infty}\frac{72\cancel{x}}{-96\cancel{x}} = \frac{72}{-96} = \frac{36}{-48} = \frac{18}{-24} = \frac{9}{-12}$$

$$= \frac{3}{-4}$$

$\frac{1}{1000}$

Recall

$$\lim_{x\to\infty}\frac{ax^m+\dots}{bx^n+\dots} = \begin{cases}\infty\ ; & m>n\\ \frac{a}{b}\ ; & m=n\\ 0\ ; & m<n\end{cases}$$

$(e^{\ln x} = x)$ $\ln(e^x) = x$

Hierarchy

$\log(x) \ll x \ll x^2 \ll x^3 \ll \ldots \ll e^x$

"P=NP"?

$\ln(a^b) = b\ln(a)$

$$\lim_{n\to\infty} \frac{2^n}{n^2}$$

Thm 5.1

$$= \lim_{x\to\infty} \frac{2^x}{x^2} \qquad \frac{\infty}{\infty}$$

L.H.

$$= \lim_{x\to\infty} \frac{\ln(2)\cdot 2^x}{2x} \qquad \frac{\infty}{\infty}$$

L.H.

$$= \lim_{x\to\infty} \frac{(\ln(2))^2\cdot 2^x}{2} \qquad \infty$$

diverges!

$$\frac{d}{dx}2^x = \frac{d}{dx}e^{\ln(2^x)}$$

$$= \frac{d}{dx}e^{x\ln(2)}$$

$$= \left(e^{x\ln(2)}\right)(\ln(2))$$

$$= \ln(2)\cdot 2^x$$

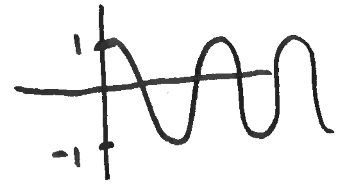
Theorem 5.4 (Squeeze thm): if there is L such that

$$\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n \quad \text{AND} \quad a_n \le c_n \le b_n,$$

THEN

$$\lim_{n\to\infty} c_n = L$$

EX: $\displaystyle\lim_{n\to\infty} \frac{\cos(n)}{n^2}$

Recall: 

i.e. $-1 \le \cos(x) \le 1$

Since

$$-1 \le \cos(n) \le 1$$

$\downarrow$ div by n^2

$$-\frac{1}{n^2} \le \frac{\cos(n)}{n^2} \le \frac{1}{n^2}$$

AND ~~BUT~~ $\displaystyle\lim_{n\to\infty} -\frac{1}{n^2} = 0 = \lim_{n\to\infty}\frac{1}{n^2}$,

therefore $\displaystyle\lim_{n\to\infty}\frac{\cos(n)}{n^2} = 0$

Ex: $\lim_{n\to\infty} \left(-\frac{1}{2}\right)^n$

$\left(-\frac{1}{2}\right)^x$

$x = 1/2$

$\downarrow$

imag #'s

$$-\frac{1}{2^n} \le \left(-\frac{1}{2}\right)^n \le \frac{1}{2^n}$$

We observe

$$\lim_{n\to\infty} \frac{1}{2^n} = 0 = \lim_{n\to\infty} \left(-\frac{1}{2^n}\right)$$

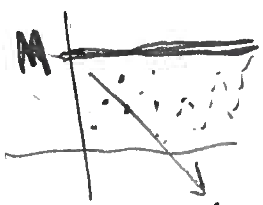
$\frac{1}{2^x}$

Therefore, $\lim_{n\to\infty} \left(-\frac{1}{2}\right)^n = 0$

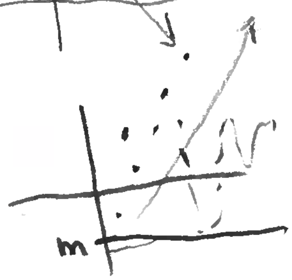
Boundedness

= "bounded"

"such that"

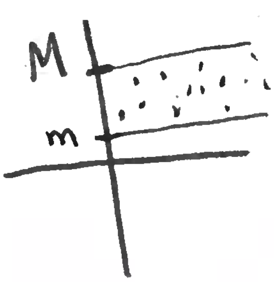


A sequence $\{a_n\}$ is bdd above provided that there is a number M s.t. $a_n \le M$ for all n.



A sequence $\{a_n\}$ is bdd below provided that there is a number m s.t. $a_n \ge m$ for all n.

A sequence $\{a_n\}$ is bounded provided it is both bdd above + bdd below.



If $\{a_n\}$ is not bdd, then we say it is unbounded.

FACTORIALS: $3! = 3\cdot2\cdot1$ $\quad 6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1$

$$\frac{(n+2)!}{(n-1)!} = \frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$$

Monotone means either increasing always or decreasing always