$$\int u\,dv = uv - \int v\,du$$

$$(uv)' = u'v + uv'$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$$\int_0^x \sim\sim\, dt = t$$

LEAD center ~ help w/ calc 2!!

Ex: $\displaystyle\int_1^3 \frac{1}{x^2+3x-4}\,dx$.

$$\frac{1}{x^2+3x-4} = \frac{1}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$$

$$0x + 1 = (A)(x-1) + (B)(x+4)$$

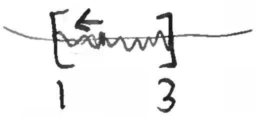
$$= (A+B)x + (-A+4B)$$

$$\begin{cases} A+B=0 \rightarrow \boxed{A=-B} \\ -A+4B=1 \end{cases}$$

$$5B = 1$$

$$B = 1/5 \quad A = -1/5$$

So, $\displaystyle\int_1^3 \frac{1}{x^2+3x-4}\,dx = \lim_{a\to1^+}\int_a^3 \frac{1}{(x+4)(x-1)}\,dx$



$\displaystyle = \lim_{a\to1^+}\left[\int_a^3 \left(-\frac{1}{5}\right)\underbrace{\frac{1}{x+4}}_{\ln(x+4)}\,dx + \int_a^3 \left(\frac{1}{5}\right)\underbrace{\frac{1}{x-1}}_{\ln(x-1)}\,dx\right]$

$\displaystyle = \lim_{a\to1^+}\left[-\frac{1}{5}\left[\ln(7)-\underbrace{\ln(a+4)}_{\ln(5)}\right] + \frac{1}{5}\left[\ln(3-1)-\underbrace{\ln(a-1)}_{\ln(0)}\right]\right]$

$-\infty$ "diverges"

= diverges to $(-\infty)$

Ex: $\displaystyle\int_1^2 \frac{1}{\sqrt{x-1}}\,dx = \lim_{a\to1^+}\int_a^2 (x-1)^{-1/2}\,dx$

problem at 1 (blows up)

$u = x-1$

$du = dx$

$\displaystyle = \lim_{a\to1^+}\int_{a-1}^1 u^{-1/2}\,du$

$\displaystyle = \lim_{a\to1^+}\left[\frac{u^{-1/2+1}}{-1/2+1}\right]_{a-1}^1$

$\displaystyle = \lim_{a\to1^+}\left.\frac{u^{1/2}}{1/2}\right|_{a-1}^1 = 2\lim_{a\to1^+}\left[1^{1/2}-\underbrace{(a-1)^{1/2}}_{\sqrt{a-1}\,\to\,0}\right]$

$= 2$

FACT $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ exists only for $0<p<1$

x^{-p+1}

Related to HW9

$$\mathcal{L}\{f\}(x) \overset{\text{def}}{=} \int_0^\infty f(t)e^{-xt}\,dt \qquad e^a e^b = e^{a+b}$$

Let $f(t) = e^{3t}$. Then

$$\mathcal{L}\{f\}(x) = \int_0^\infty e^{3t}e^{-xt}\,dt \qquad e^{3t-xt}$$

$u = (3-x)t$

$du = (3-x)\,dt$

$\frac{1}{3-x}\,du = dt$

$$= \lim_{b\to\infty} \int_0^b e^{(3-x)t}\,dt$$

$$= \lim_{b\to\infty} \frac{1}{3-x}\int_0^{(3-x)b} e^u\,du \quad \text{antideriv is } e^u$$

$$\lim_{b\to\infty} e^{(3-x)b} = \begin{cases} 0, & 3-x<0 \\ 1, & x=3,\ 3-x=0 \\ \infty, & 3-x>0 \end{cases}$$

$$= \lim_{b\to\infty} \frac{1}{3-x}\left[e^{(3-x)b} - 1\right]$$

$x>3$

$0>3-x$

$3-x<0$

$$(3-x<0) \quad = \frac{1}{3-x}[0-1]$$

$$= \frac{1}{x-3}$$