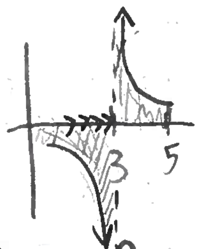
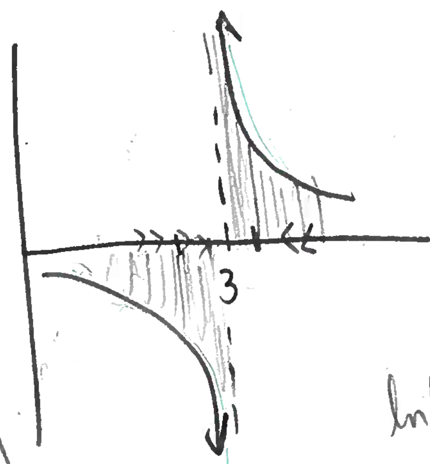
Ex: $\displaystyle\int_0^5 \frac{1}{x^2-2x-3}\,dx = \int_0^3 \frac{1}{x^2-2x-3}\,dx + \int_3^5 \frac{1}{x^2-2x-3}\,dx$

$(x-3)(x+1)$

$\displaystyle = \lim_{b\to 3^-}\int_0^b \frac{1}{(x-3)(x+1)}\,dx + \lim_{a\to 3^+}\int_a^5 \frac{1}{(x-3)(x+1)}\,dx$





$\displaystyle \frac{1}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$

$0x + 1 = (A)(x+1) + (B)(x-3)$

$= (A+B)x + (A-3B)$

$\Rightarrow \begin{cases} A+B=0 \rightarrow \boxed{A=-B} \\ A-3B=1 \rightarrow -4B=1 \end{cases}$

$\boxed{B=-1/4}$

$\boxed{A=1/4}$

$\displaystyle\int \frac{1}{x}\,dx = \ln(|x|)$

control theory

$\ln|x-3|$

$\displaystyle = \lim_{b\to 3^-}\left[\frac{1}{4}\int_0^b \frac{1}{x-3}\,dx - \frac{1}{4}\int_0^b \frac{1}{x+1}\,dx\right]$

$\ln(x-3)$

$\displaystyle + \lim_{a\to 3^+}\left[\frac{1}{4}\int_a^5 \frac{1}{x-3}\,dx - \frac{1}{4}\int_a^5 \frac{1}{x+1}\,dx\right]$

$\to \ln(0)$ as $b\to 3^-$ so it DNE

$\displaystyle = \lim_{b\to 3^-}\left[\frac{1}{4}\left(\ln(b-3) - \ln(|-3|)\right) - \frac{1}{4}\left(\ln(b+1) - \ln(1)\right)\right]$

$\displaystyle + \lim_{a\to 3^+}\left[\frac{1}{4}\left(\ln(2) - \ln(a-3)\right) - \frac{1}{4}\left[\ln(6) - \ln(a+1)\right]\right]$

$\ln(0)$ as $a\to 3^+$ so it DNE

Ex: $\int_0^1 \frac{\ln(x)}{\sqrt{x}}\,dx = \lim_{a\to 0^+}\int_a^1 \frac{\ln(x)}{\sqrt{x}}\,dx$

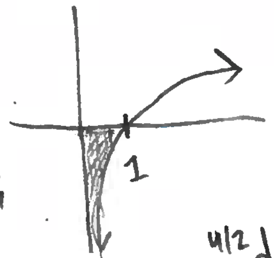
$u=\ln(x)$

$du=\frac{1}{x}dx$

$= \lim_{a\to 0^+}\int_a^1 \frac{\sqrt{x}\ln(x)}{x}\,dx$

$e^u = x$

$e^{u/2}=\sqrt{x}$



L'Hôpital

$\lim \frac{f}{g} = \lim \frac{f'}{g'}$

if 1st limit is indet.

$= \lim_{a\to 0^+}\int_{\ln(a)}^{0} e^{u/2}\,u\,du$

$\xi = \frac{u}{2}$

$2d\xi = du$

$w=u \quad dv=e^{u/2}du$

$dw=du \quad v=2e^{u/2}$

$\alpha\beta \quad \alpha\ln(\beta)=\ln(\beta^{\alpha})$

$= \lim_{a\to 0^+}\left[u\cdot 2e^{u/2}\Big|_{\ln(a)}^{0} - 2\int_{\ln(a)}^{0} e^{u/2}\,dw\right]$

$\frac{\ln(a)}{2} = \ln(a^{1/2})$ $\quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}$

$= \lim_{a\to 0^+}\left[\left(0-\ln(a)2\cdot e^{\frac{\ln(a)}{2}}\right) - \left[4e^{u/2}\Big|_{\ln(a)}^{0}\right]\right]$

$= \lim_{a\to 0^+}\left[\left(\cancel{-\ln(a)2\sqrt{a}}\right)^{0} - \left[4(1)-4\cancel{\sqrt{a}}^{\,0}\right]\right]$

$= -4$

$\log(x) \ll \sqrt{x} \ll x$

Q: What is $\lim_{a\to 0^+}\ln(a)\sqrt{a}$?

$\ln(0)\sqrt{0}$

$(-\infty)(0)$

$\ln(a)\sqrt{a} = \frac{\sqrt{a}}{\frac{1}{\ln(a)}}$

$= \frac{\ln(a)}{\frac{1}{\sqrt{a}}}$

$= \lim_{a\to 0^+}\frac{\frac{d}{da}\ln(a)}{\frac{d}{da}a^{-1/2}}$

$= \lim_{a\to 0^+}\frac{1/a}{(-1/2)a^{-3/2}} = (-2)\lim_{a\to 0^+}\frac{1}{a}\cdot a^{3/2} = -2\lim_{a\to 0^+}\sqrt{a} = 0$

$a^{-3/2} = \frac{1}{a^{3/2}}$

Ex: $\displaystyle\int_0^1 \frac{\ln(x)}{x}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{\ln(x)}{x}\,dx$

$u = \ln(x)$
$du = \frac{1}{x}\,dx$

$$= \lim_{a\to 0^+} \int_{\ln(a)}^{0} u\,du$$

$$= \lim_{a\to 0^+} \left.\frac{u^2}{2}\right|_{\ln(a)}^{0}$$

$$= \lim_{a\to 0^+} -\frac{\ln(a)^2}{2}$$

$$= -\frac{1}{2}\Big(\underbrace{\lim_{a\to 0^+} \ln(a)}_{-\infty}\Big)^2$$

$$= -\infty \quad \text{DNE} \quad (\text{"divergent integral"})$$

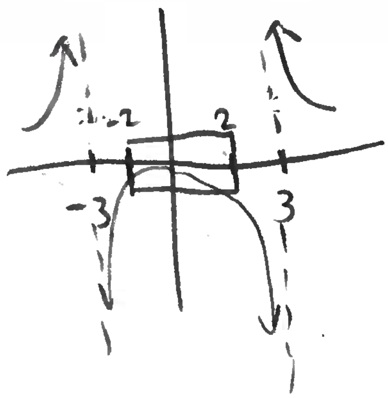
Turns out

$\displaystyle\int_0^1 \frac{\ln(x)}{x^p}\,dx$ exists only when $0 < p < 1$

(see "L^p spaces")

L^2 spaces $\leadsto$ quantum mech.

$$\int_{-2}^{2} \frac{1}{x^2-9}\,dx = \int_{-2}^{2} \frac{1}{(x-3)(x+3)}\,dx$$



$$= \frac{1}{6}\int_{-2}^{2}\frac{1}{x-3}\,dx + \left(-\frac{1}{6}\right)\int_{-2}^{2}\frac{1}{x+3}\,dx$$

$$= \frac{1}{6}\ln(|x-3|)\Big|_{-2}^{2} - \frac{1}{6}\ln|x+3|\Big|_{-2}^{2}$$

$$= \frac{1}{6}\left[\ln(1)-\ln(5)\right] - \frac{1}{6}\left[\ln(5)-\ln(1)\right]$$

$$= -\frac{1}{3}\ln(5)$$

$$\frac{A}{x-3} + \frac{B}{x+3} = \frac{1}{(x-3)(x+3)}$$

$$(A)(x+3) + (B)(x-3) = 0x + 1$$

$$(A+B)x + (3A-3B) = 0x + 1$$

$$\begin{cases} A+B=0 \rightarrow B=-A \\ 3A-3B=1 \rightarrow 6A=1 \end{cases} \quad \boxed{A=1/6} \quad \boxed{B=-1/6}$$

Webwork

$|x|$

abs(x)

$(x-3)(x+3)$