Continued from 1 March

We arrived at

$u = x+5$ $\quad u = x^2+4$ $\quad \frac{1}{2}du = x\,dx$

$$\int_{-1}^{2} \frac{1}{(x+5)(x^2+4)}dx = \frac{1}{29}\int_{-1}^{2}\frac{1}{x+5}dx - \frac{1}{29}\int_{-1}^{2}\frac{x}{x^2+4}dx$$

$$+\frac{5}{29}\int_{-1}^{2}\frac{1}{x^2+4}dx$$

$(x^2+4)\frac{1}{4}$

$= \frac{x^2}{4}+1$

$= \left(\frac{x}{2}\right)^2+1$

$$= \frac{1}{29}\ln(x+5)\Big|_{-1}^{2} - \frac{1}{58}\int_{5}^{8}\frac{1}{u}du$$

$u = \frac{x}{2}$

$2du = dx$

$$+\frac{5}{116}\int_{-1}^{2}\frac{1}{\left(\frac{x}{2}\right)^2+1}dx$$

$\ln(a)-\ln(b) = \ln\left(\frac{a}{b}\right)$

anti-deriv is arctan(u)

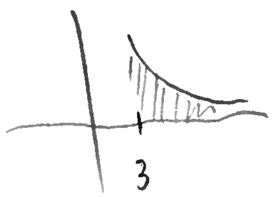
$$= \frac{1}{29}\left[\ln(7)-\ln(4)\right] - \frac{1}{58}\ln(u)\Big|_{5}^{8} + \frac{5}{58}\int_{-1/2}^{1}\frac{1}{u^2+1}du$$

$$= \frac{1}{29}\left[\ln\left(\frac{7}{4}\right)\right] - \frac{1}{58}\ln\left(\frac{8}{5}\right) + \frac{5}{58}\left[\arctan(1) - \arctan\left(-\frac{1}{2}\right)\right]$$

Improper integrals

Types:

(A) integrals over an infinite domain



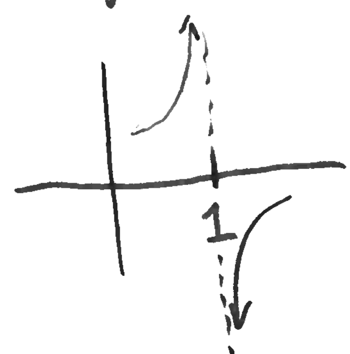
$$\int_a^{\infty} \quad \text{or} \quad \int_{-\infty}^{a} \quad \text{or} \quad \int_{-\infty}^{\infty}$$

Laplace transform

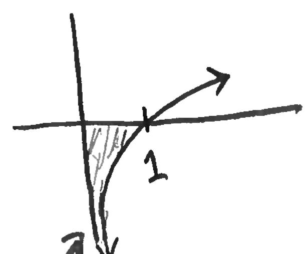
Fourier analysis

(B) integrals with functions who have an asymptote ("place it blows up") on its region of integration

$$\int_0^2 \frac{1}{1-x}\,dx$$



$$\int_0^1 \frac{\ln(x)}{\sqrt{x}}\,dx$$



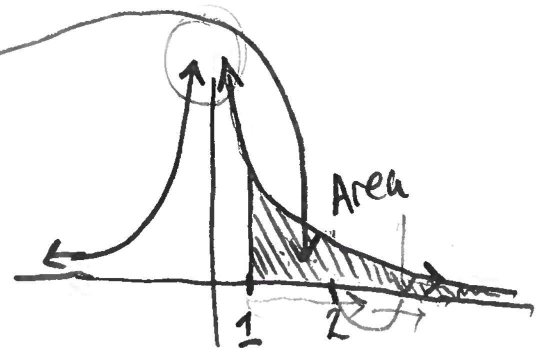
trouble at 1

trouble at 0

Both types get resolved with limits!

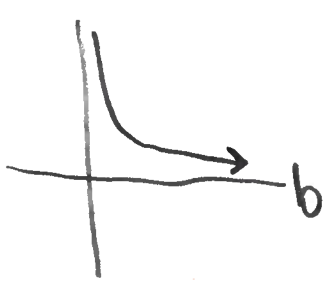
Example: How do we handle something like

$$\int_1^\infty \frac{1}{x^2}\,dx \;?$$



We use limits:

$$\int_1^\infty x^{-2}\,dx \overset{\text{define}}{=} \lim_{b\to\infty} \int_1^b x^{-2}\,dx \qquad \frac{1}{1^2} \to \frac{1}{2^2}$$



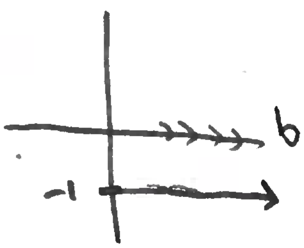
$$= \lim_{b\to\infty} \left.\frac{x^{-1}}{-1}\right|_1^b \qquad \lim_{x\to\infty} \frac{P(x)}{Q(x)} = 0$$

polynom

wherever $\deg(Q(x)) > \deg(P(x))$

$$= -\lim_{b\to\infty}\left(\frac{1}{b} - 1\right)$$

$\frac{1}{b} \to 0$



$$= -(0-1) = 1$$

Hierarchy of function growth

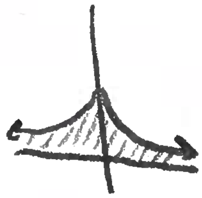
$$\lim_{x\to\infty} \frac{x^{500}+x^2+5}{e^x} = 0 \qquad \log x \ll x \ll x^2 \ll x^3 \ll \dots \ll e^x \qquad \lim_{x\to\infty} \frac{5x^2+3x+1}{6x^2-x+3}$$

$$= \frac{5}{6}$$

$$\lim_{x\to\infty} \frac{x^2+1}{\ln(x)} = \infty$$

$x\log(x)$

Ex: $\int_{-\infty}^{\infty} e^{-|x|}\,dx \overset{\text{define}}{=} \int_{0}^{\infty} e^{-|x|}\,dx + \int_{-\infty}^{0} e^{-|x|}\,dx$



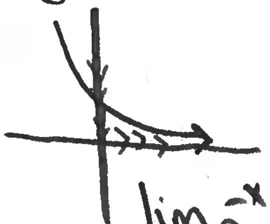
$= \lim_{b\to\infty} \int_{0}^{b} e^{-x}\,dx + \lim_{a\to\infty} \int_{-a}^{0} e^{x}\,dx$

$u = -x$

$-du = dx$

$= \lim_{b\to\infty} -e^{-x}\Big|_{0}^{b} + \lim_{a\to\infty} e^{x}\Big|_{-a}^{0}$

e^{-x}



$= \lim_{b\to\infty} \left(\underbrace{\cancel{-e^{-b}}}_{0} + e^{0}\right) + \lim_{a\to\infty} \left[e^{0} - \underbrace{\cancel{e^{-a}}}_{0}\right]$

$\lim_{x\to\infty} e^{-x} = 0$

$= (0+1) + (1-0)$

$= 2$