Last time: (had bad bounds... ignore!)

$$\int \frac{5x^2}{\sqrt{x^2-1}}\,dx = 5\int \sec^3(\theta)\,d\theta$$

$$= 5\sec(\theta)\tan(\theta) - \int \sec^3(\theta)\,d\theta + \int \sec(\theta)\,d\theta$$

$$6\int \sec^3(\theta)\,d\theta = 5\sec(\theta)\tan(\theta) + \ln(\tan(\theta)+\sec(\theta)) + C$$

$$\int \sec^3(\theta)\,d\theta = \frac{5}{6}\sec(\theta)\tan(\theta) + \frac{1}{6}\ln(\tan(\theta)+\sec(\theta)) + C$$

$$\int \frac{5x^2}{\sqrt{x^2-1}}\,dx = 5\int \sec^3(\theta)\,d\theta$$

$$= \frac{25}{6}\sec(\theta)\tan(\theta) + \frac{5}{6}\ln(\tan(\theta)+\sec(\theta)) + C$$

2

Ex:

$$\int \frac{1}{\sqrt{x^2-9}}\,dx \qquad 9 = 3^2$$

$$x = 3\sec(\theta)$$

$$dx = 3\tan(\theta)\sec(\theta)\,d\theta$$

$$\frac{d}{dx}\frac{1}{\cos x} = \frac{+\sin x}{\cos^2 x} = \tan(x)\sec(x)$$

$$= \int \frac{1}{\sqrt{9\sec^2(\theta)-9}}\,3\tan(\theta)\sec(\theta)\,d\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$= \int \frac{1}{\cancel{\sqrt{\sec^2(\theta)-1}}}\,\cancel{\tan(\theta)}\sec(\theta)\,d\theta$$

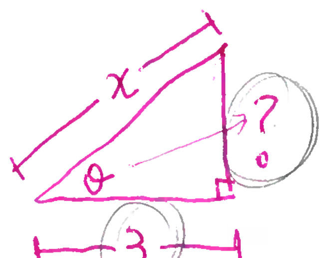
$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$\sqrt{\sec^2(\theta)-1} = \sqrt{\tan^2(\theta)}$$

$$= \int \sec(\theta)\,d\theta$$

$$\sec(\theta) = \frac{x}{3} \qquad \sec \approx \frac{hyp}{adj}$$

$$= \ln\left(\tan(\theta) + \underbrace{\sec(\theta)}_{=\frac{x}{3}}\right) + C$$



$$3^2 + ?^2 = x^2 \rightarrow ? = \sqrt{x^2-9}$$

$$= \ln\left(\frac{\sqrt{x^2-9}}{3} + \frac{x}{3}\right) + C$$

like #5

$$\int \frac{7x^2}{\sqrt{4-x^2}}\,dx$$

$4 = 2^2$

$x = 2\sin(\theta) \rightarrow x^2 = 4\sin^2\theta$

$\downarrow$

$dx = 2\cos(\theta)\,d\theta$

③

$\frac{d}{d\theta}\cos^2\theta$
$\shortparallel$
$-2\cos\theta\sin\theta$

$$= 7\int \frac{4\sin^2(\theta)}{\sqrt{4-4\sin^2(\theta)}}\,2\cos(\theta)\,d\theta$$

$\sin^2\theta + \cos^2\theta = 1$

$\sqrt{\cos^2\theta} = \sqrt{1-\sin^2\theta}$

$\frac{d}{d\theta}\sin^3(\theta)$
$\shortparallel$
$3\sin^2\theta\cos\theta$

$$= 28\int \frac{\sin^2(\theta)\cos(\theta)}{\sqrt{1-\sin^2(\theta)}}\,d\theta$$

$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$

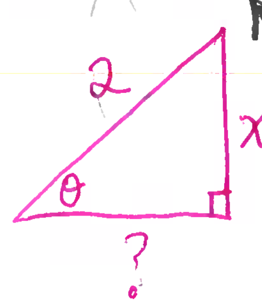
$\sin(2x) = 2\sin(x)\cos(x)$

$$= 28\int \sin^2(\theta)\,d\theta$$

$$= 14\int 1-\cos(2\theta)\,d\theta$$

$\sin(\theta) = \frac{x}{2} \leftarrow \cancel{\theta = \frac{x}{2\sin}}$

$$= 14\left[\theta - \frac{1}{2}\sin(2\theta)\right] + C$$



$\theta = \arcsin\left(\frac{x}{2}\right)$

$$= 14\theta - 14\sin(\theta)\cos(\theta) + C$$

$$= 14\arcsin\left(\frac{x}{2}\right) - 14\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C$$

$?^2 + x^2 = 4$

$? = \sqrt{4-x^2}$

Ex: $\displaystyle\int \frac{7}{\sqrt{x^2+16}}\,dx$ $\quad 16 = 4^2$ $\qquad \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{c^2+s^2}{c^2} = \frac{1}{c^2}$

$x = 4\tan(\theta)$

$dx = 4\sec^2(\theta)$

$$= \int \frac{7}{\sqrt{16\tan^2(\theta)+16}}\, 4\sec^2(\theta)\,d\theta$$

$\sin^2\theta + \cos^2\theta = 1$

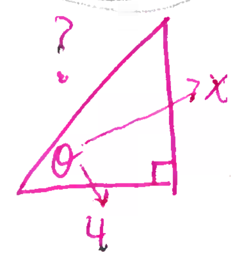
$\downarrow$

$\sqrt{\tan^2\theta + 1} = \sqrt{\sec^2\theta}$

$$= 7\int \frac{1}{\sqrt{\tan^2(\theta)+1}}\sec^2(\theta)\,d\theta$$

$$= 7\int \sec(\theta)\,d\theta$$

$\tan\theta = \frac{x}{4}$



$$= 7\ln(\tan(\theta) + \sec(\theta)) + C$$

$$= 7\ln\left(\frac{x}{4} + \frac{\sqrt{16+x^2}}{4}\right) + C$$

$4^2 + x^2 = ?^2$

$? = \sqrt{16+x^2}$

$$\int \frac{x}{\sqrt{x^4+1}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{u^2+1}}\,du$$

$u = \tan\theta$

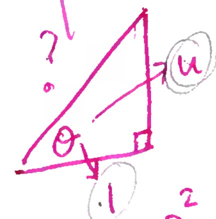
$du = \sec^2\theta\, d\theta$

$(x^2)^2 = x^4$

$u = x^2$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int \frac{1}{\cancel{\sqrt{\tan^2\theta+1}}}\sec^{\cancel{2}}\theta\, d\theta$$



$$= \frac{1}{2}\int \sec(\theta)\,d\theta$$

$1^2+u^2 = ?^2 \rightarrow$

$? = \sqrt{1+u^2}$

$$= \frac{1}{2}\ln\left(\tan(\theta)+\sec(\theta)\right)+C$$

$$= \frac{1}{2}\ln\left(u+\sqrt{1+u^2}\right)+C$$

$$= \frac{1}{2}\ln\left(x^2+\sqrt{1+x^4}\right)+C$$