If your $\int$ contains... make substitution

$$\sqrt{a^2-x^2} \longrightarrow x = a\sin(\theta)$$

$$\sqrt{a^2+x^2} \longrightarrow x = a\tan(\theta)$$

$$\sqrt{x^2-a^2} \longrightarrow x = a\sec(\theta)$$

Recall:

$$\int \csc(\theta)\,d\theta = -\ln(\cot(\theta)+\csc(\theta)) + C$$

$$\int \sec(\theta)\,d\theta = \ln(\tan(\theta)+\sec(\theta)) + C$$

Ex: $$\int_0^{15} \frac{1}{\sqrt{9+x^2}}\,dx = \int_{x=0}^{x=15} \frac{1}{\sqrt{9+9\tan^2(\theta)}}\,3\sec^2(\theta)\,d\theta$$

$9 = 3^2$

$a = 3$

$x = 3\tan(\theta) \leftarrow \frac{x}{3} = \tan(\theta) \leftarrow \theta = \arctan\left(\frac{x}{3}\right)$

$dx = 3\sec^2(\theta)\,d\theta$

$$\frac{d}{dx}\tan(x) = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$\sin^2\theta + \cos^2(\theta) = 1$

$\sqrt{\tan^2\theta + 1} = \sqrt{\sec^2(\theta)}$

$$= \int_{x=0}^{x=15} \frac{\sec^2(\theta)}{\sqrt{1+\tan^2(\theta)}}\,d\theta$$

$$= \int_{x=0}^{x=15} \sec(\theta)\,d\theta = \ln\left(\tan(\theta)+\sec(\theta)\right)\Big|_{x=0}^{x=15}$$

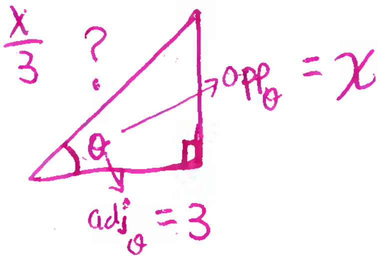
$$= \ln\left(\cancel{\tan(\arctan}(\tfrac{x}{3})) + \sec(\arctan(\tfrac{x}{3}))\right)\Big|_{x=0}^{x=15}$$

$$= \ln\left(\frac{x}{3} + \sec\left(\arctan\left(\frac{x}{3}\right)\right)\right)\Bigg|_{x=0}^{x=15}$$

Evaluate

Let $\theta = \arctan\left(\frac{x}{3}\right)$

$$\frac{\text{opp}}{\text{adj}} = \tan(\theta) = \frac{x}{3}$$



Pyth thm:

$$3^2 + x^2 = ?^2$$

$$? = \sqrt{x^2+9}$$

$$\Rightarrow \boxed{\sec\left(\arctan\left(\frac{x}{3}\right)\right) = \sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{x^2+9}}{3}}$$

$$= \ln\left(\frac{x}{3} + \frac{\sqrt{x^2+9}}{3}\right)\Bigg|_0^{15}$$

$$= \underbrace{\ln\left(5 + \frac{\sqrt{9+15^2}}{3}\right)} - \ln(1)$$

$$\underline{\text{Ex}}: \int_0^1 \frac{3x}{\sqrt{1-x^2}}\,dx = -\frac{3}{2}\int_1^0 u^{-1/2}\,du$$

$u = 1-x^2$

$-\frac{1}{2}du = x\,dx$

$$= -\frac{3}{2}\,\frac{u^{1/2}}{1/2}\Bigg|_1^0$$

$$= -\frac{3}{\cancel{2}}\left[0 - 1\right]$$

$$= 3$$

$$\underline{\text{Ex}}: \int_0^1 \frac{3x}{\sqrt{1-x^2}}\,dx = \int_{x=0}^{x=1} \frac{3\sin(\theta)}{\sqrt{1-\cos^2(\theta)}}\cos(\theta)\,d\theta$$

$x = \sin(\theta)$

$dx = \cos(\theta)\,d\theta$

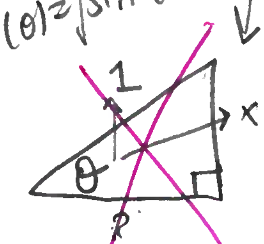
$$= 3\int_{x=0}^{x=1} \cos(\theta)\,d\theta$$

$$= 3\sin(\theta)\Big|_{x=0}^{x=1}$$

$$= 3x\Big|_{x=0}^{x=1} = 3-0 = 3$$

$\cos^2\theta + \sin^2\theta = 1$

$\sqrt{1-\cos^2(\theta)} = \sqrt{\sin^2(\theta)}$



unnecessary!

Ex: $\int_{0}^{1/2} \frac{5x^2}{\sqrt{x^2-1}}\,dx = \int_{x=0}^{x=1/2} \frac{5\sec^2(\theta)}{\sqrt{\sec^2(\theta)-1}} \sec(\theta)\cancel{\tan(\theta)}\,d\theta$

$$\boxed{\begin{aligned} x &= \sec(\theta) \\ dx &= \sec(\theta)\tan(\theta)\,d\theta \end{aligned}}$$

$\cancel{x = \sqrt{u+1}}$

$\cancel{u = x^2 - 1}$

$\cancel{\tfrac{1}{2}\,du = x\,dx}$

$\cancel{\frac{5\sqrt{u+1}}{\sqrt{u}}}$

$\cancel{5\sqrt{1+\frac{1}{u}}}$

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\boxed{1 + \tan^2(\theta) = \sec^2\theta}$

$\sqrt{\sec^2(\theta) - 1} = \sqrt{\tan^2(\theta)}$

$\frac{d}{dx}\frac{1}{\cos x} = \frac{0 + \sin x}{\cos^2 x} = \tan(x)\sec(x)$

$$= \boxed{5\int_{x=0}^{x=1/2} \sec^3(\theta)\,d\theta}$$

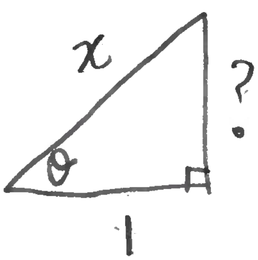
$u = \sec\theta \quad dv = \sec^2\theta$

$du = \sec\theta\tan\theta\,d\theta \quad v = \tan\theta$

$$= 5\sec(\theta)\tan(\theta)\Big|_{x=0}^{x=1/2} - \int_{x=0}^{x=1/2} \sec(\theta)\boxed{\tan^2(\theta)}\,d\theta$$

$\leftarrow \sec^2(\theta) - 1$

$$= 5\sec(\theta)\tan(\theta)\Big|_{x=0}^{x=1/2} - \boxed{\int_{x=0}^{x=1/2} \sec^3(\theta)\,d\theta} + \int_{x=0}^{x=1/2} \sec(\theta)\,d\theta$$



$1^2 + ?^2 = x^2$

$? = \sqrt{x^2-1}$

$$6\int_{x=0}^{x=1/2} \sec^3(\theta)\,d\theta = 5\sec(\theta)\tan(\theta)\Big|_{x=0}^{x=1/2} + \ln\big(\tan(\theta) + \sec(\theta)\big)\Big|_{x=0}^{x=1/2}$$

$$= 5x\sqrt{x^2-1}\,\Big|_0^{1/2} + \ln\left(\sqrt{x^2-1} + x\right)\Big|_0^{1/2}$$

$$= \frac{5}{2}$$

TRY AGAIN

TOMORROW