Trig integrals

reduction formulas

Recall

$\star\ \cos^2(x)+\sin^2(x)=1$

$\star\ \sin^2(t)=\dfrac{1-\cos(2t)}{2}$

$\star\ \cos^2(t)=\dfrac{1+\cos(2t)}{2}$

$$\int \cos^j(x)\sin^k(x)\,dx$$

Method

1. if k odd $\rightarrow$ rewrite

$\sin^k(x)=\sin(x)\sin^{k-1}(x)$ ← even

Use $\sin^2(t)=1-\cos^2(t)$ numerous times

2. if j odd $\rightarrow$ rewrite

$\cos^j(x)=\cos(x)\cos^{j-1}(x)$

~~reduction formula~~ pyth identity

3. j, k both even

↳ use reduction formulas

$$(a^b)^c = a^{bc}$$

Ex: $\int \cos^8(x)\sin^5(x)\,dx$

$$\sin^5(x) = \sin(x)\sin^4(x)$$
$$= \sin(x)\left(\sin^2(x)\right)^2$$
$$= \sin(x)\underbrace{\left(1-\cos^2(x)\right)^2}$$

$$= \int \cos^8(x)\left[1 - 2\cos^2(x) + \cos^4(x)\right]\sin(x)\,dx$$

$$= \int \cos^8(x)\sin(x)\,dx - 2\int \cos^{10}(x)\sin(x)\,dx + \int \cos^{12}(x)\sin(x)\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$= -\int u^8\,du + 2\int u^{10}\,du - \int u^{12}\,du$$

$$= -\frac{u^9}{9} + \frac{2u^{11}}{11} - \frac{u^{13}}{13} + C$$

$$= -\frac{\cos^9(x)}{9} + \frac{2\cos^{11}(x)}{11} - \frac{\cos^{13}(x)}{13} + C$$

$\cos^2 x + \boxed{\sin^2 x} = 1$

$\sin^2(x) = 1 - \cos^2(x)$

Ex: $\int \cos^3(x)\sin(x)\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= -\int u^3\,du$

$= -\frac{u^4}{4} + C$

$= \frac{-\cos^4 x}{4} + C$

Ex: $\int \cos^2(x)\sin^3(x)\,dx$

$\sin^3(x)$

$= \boxed{\sin^2(x)}\sin(x)$

$= (1-\cos^2(x))\sin(x)$

$= \int \cos^2(x)(1-\cos^2(x))\sin(x)\,dx$

$= \int \cos^2(x)\sin(x)\,dx - \int \cos^4(x)\sin(x)\,dx$

$u = \cos x$

$du = -\sin(x)\,dx$

$= -\int u^2\,du + \int u^4\,du = \frac{-u^3}{3} + \frac{u^5}{5} + C$

$= \frac{-\cos^3(x)}{3} + \frac{\cos^5(x)}{5} + C$

Ex: $\int \cos^2(x)\sin^2(x)\,dx$

$$= \int \left(\frac{1+\cos(2x)}{2}\right)\left(\frac{1-\cos(2x)}{2}\right)dx$$

$$= \frac{1}{4}\int 1 - \boxed{\cos^2(2x)}\,dx \qquad = \frac{1+\cos(4x)}{2}$$

$$= \frac{1}{4}\left[\int 1 - \frac{1+\cos(4x)}{2}\,dx\right]$$

$$= \frac{1}{4}\left[\int \frac{1}{2}\,dx - \frac{1}{2}\int \cos(4x)\,dx\right]$$

$$= \frac{1}{4}\left[\frac{1}{2}x - \frac{1}{8}\sin(4x) + C\right]$$

Ex: $\int \sin^4(x)\,dx$

reduce order

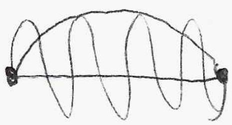
$$\cos^2(2x) = \frac{1+\cos(4x)}{2}$$

Soln: $\sin^4(x) = \left(\sin^2(x)\right)^2$

reduction of order $\rightarrow$ $= \left(\dfrac{1-\cos(2x)}{2}\right)^2$

$$= \frac{1-2\cos(2x)+\boxed{\cos^2(2x)}}{4}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}$$



$$= \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{\frac{1+\cos(4x)}{2}}{4/1}$$

$$= \frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{\cos(4x)}{8}$$

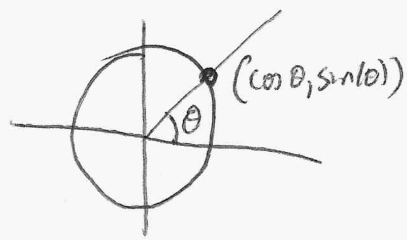
So, $\int \sin^4(x)\,dx = \int \frac{3}{8}\,dx - \frac{1}{2}\int \cos(2x)\,dx + \frac{1}{8}\int \cos(4x)\,dx$

$$= \frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$$

$\cos^2(x)+\sin^2(x)=1$

$\cos^2(x)=1-\sin^2(x)$

$x^2+y^2=1$



EX: $\int \boxed{\cos^3(x)} \sin^{12}(x)\,dx$

$$\cos^3(x)=\cos(x)\cos^2(x)$$
$$=\cos(x)\left(1-\sin^2(x)\right)$$

$$=\int \left(1-\sin^2(x)\right)\left(\cos(x)\sin^{12}(x)\right)dx$$

$u=\sin(x)$

$du=\cos(x)\,dx$

$$=\int \sin^{12}(x)\cos(x)\,dx-\int \sin^{14}(x)\cos(x)\,dx$$

$$=\int u^{12}\,du-\int u^{14}\,du$$

$$=\frac{u^{13}}{13}-\frac{u^{15}}{15}+C=\frac{\sin^{13}(x)}{13}-\frac{\sin^{16}(x)}{16}+C$$