Ex: $\int \arcsin(x)\,dx$

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$u = \arcsin x \rightarrow du = \frac{1}{\sqrt{1-x^2}}\,dx$

$dv = 1dx \rightarrow v = x$

$$\int \underbrace{\arcsin(x)}_{u}\,\underbrace{dx}_{dv} = x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$w = 1-x^2$

$\frac{1}{2}dw = x\,dx$

$$= x\arcsin(x) - \frac{1}{2}\int w^{-1/2}\,dw$$

$$= x\arcsin(x) - \cancel{\frac{1}{2}}\,\frac{w^{1/2}}{\cancel{1/2}} + C$$

$$= x\arcsin(x) - \sqrt{1-x^2} + C$$

Ex: $\int \arcsin(7x)\,dx = \frac{1}{7}\int \arcsin(u)\,du$

$u = 7x$

$\frac{1}{7}du = dx$

$$= \frac{1}{7}\left[u\arcsin(u) - \sqrt{1-u^2} + C\right]$$

$u = 7x$

$u^2 = (7x)^2 = 49x^2$

$$= \frac{1}{7}\left[7x\arcsin(7x) - \sqrt{1-49x^2} + C\right]$$

$$= x\arcsin(7x) - \frac{1}{7}\sqrt{1-49x^2} + \tilde{C}\ ;\ \tilde{C} = \frac{1}{7}C$$

Ex: $\int e^x \cos(x)\,dx$

$u = e^x \rightarrow du = e^x\,dx$

$dv = \cos(x)\,dx \rightarrow v = \sin(x)$

$\int$ by parts again

So compute

(*) $$\int e^x \cos(x)\,dx = e^x \sin(x) - \boxed{\int \sin(x) e^x\,dx}$$

$u_2 = e^x \rightarrow du_2 = e^x\,dx$

$dv_2 = \sin(x)\,dx \rightarrow v_2 = -\cos(x)$

So compute

$$\int \sin(x) e^x\,dx = -e^x \cos(x) - \int (-\cos(x)) e^x\,dx$$

By (*):

$$\underline{\int e^x \cos(x)\,dx} = e^x \sin(x) + e^x \cos(x) - \underline{\int e^x \cos(x)\,dx}$$

$$2\int e^x \cos(x)\,dx = e^x[\sin(x) + \cos(x)]$$

Therefore,

$$\int e^x \cos(x)\,dx = \frac{e^x}{2}[\sin(x) + \cos(x)] + C$$

Ex: $\int_0^2 te^{-t}\,dt$

$\int e^{bt}\,dt = \frac{1}{b}e^{bt} + C$

Soln: $u = t \longrightarrow du = dt$

$dv = e^{-t}dt \rightarrow v = -e^{-t}$

So compute

$$\int_0^2 te^{-t}dt = -te^{-t}\Big|_0^2 + \int_0^2 e^{-t}\,dt$$

$$= \left(-2e^{-2} - 0\right) + (-1)e^{-t}\Big|_0^2$$

$$= -2e^{-2} - e^{-2} + 1$$

$$= \frac{-3}{e^2} + 1$$

Ex: Suppose that $f(2)=5$, $f(7)=3$, $f'(2)=1$, and $f'(7)=8$, and f'' is ctn.

$$\int_2^7 f''(x)\,dx = f'(2)-f'(7)$$

Compute $\displaystyle\int_2^7 x f''(x)\,dx = x f'(x)\Big|_2^7 - \int_2^7 f'(x)\,dx$

$u=x \qquad dv=f''(x)$

$du=dx \qquad v=f'(x)$

$$= \big(7f'(7)-2f'(2)\big) - f(x)\Big|_2^7$$

$$= \big(7(8)-2(1)\big) - \big(f(7)-f(2)\big)$$

$$= (56-2)-(3-5)$$

$$= 54-(-2)$$

$$= 56$$

Ex:
$$\int x^2\cos(7x)\,dx$$

$u = x^2 \rightarrow du = 2x\,dx$

$dv = \cos(7x)\,dx \rightarrow v = \frac{1}{7}\sin(7x)$

So compute

(★) $$\int x^2\cos(7x)\,dx = \frac{x^2}{7}\sin(7x) - \int \underbrace{\frac{1}{7}\sin(7x)\,2x\,dx}$$

$$= \frac{x^2}{7}\sin(7x) - \frac{2}{7}\int x\sin(7x)\,dx$$

$u_2 = x \rightarrow du_2 = dx$

$dv_2 = \sin(7x) \rightarrow v_2 = -\frac{1}{7}\cos(7x)$

So,

$$\int x\sin(7x)\,dx = x\left(-\frac{1}{7}\cos(7x)\right) - \int\left(-\frac{1}{7}\cos(7x)\right)dx$$

$$= -\frac{1}{7}x\cos(7x) + \frac{1}{7}\underbrace{\int\cos(7x)\,dx}_{\frac{1}{7}\sin(7x)}$$

So by (★)

$$\int x^2\cos(7x)\,dx = \boxed{\frac{x^2}{7}\sin(7x) - \frac{2}{7}\left[-\frac{1}{7}x\cos(7x) + \frac{1}{49}\sin(7x)\right]}$$