Ex: $f(x) = \int_1^x \sqrt{t^2-1}\, dt$

Find arclength of f on $[1,6]$

Soln: $f'(x) = \frac{d}{dx}\int_1^x \sqrt{t^2-1}\, dt = \sqrt{x^2-1}$

Fundamental thm of calc

$$\sqrt{1+[f'(x)]^2} = \sqrt{1+\left(\sqrt{x^2-1}\right)^2}$$

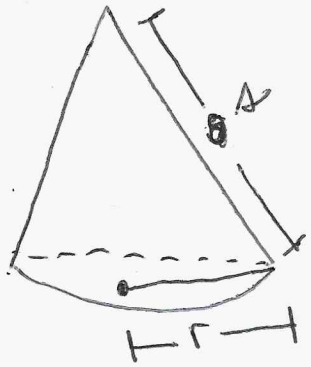
$$= \sqrt{1+x^2-1} = \sqrt{x^2} = x$$

$$\text{Arclength} = \int_1^6 \sqrt{1+(f'(x))^2}\, dx = \int_1^6 x\, dx = \frac{x^2}{2}\Big|_1^6$$

$$= \frac{36}{2} - \frac{1}{2}$$

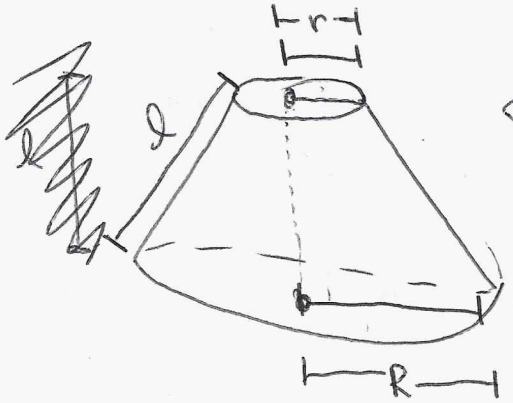
$$= \frac{35}{2}$$

Surface Area of Solids of Rev

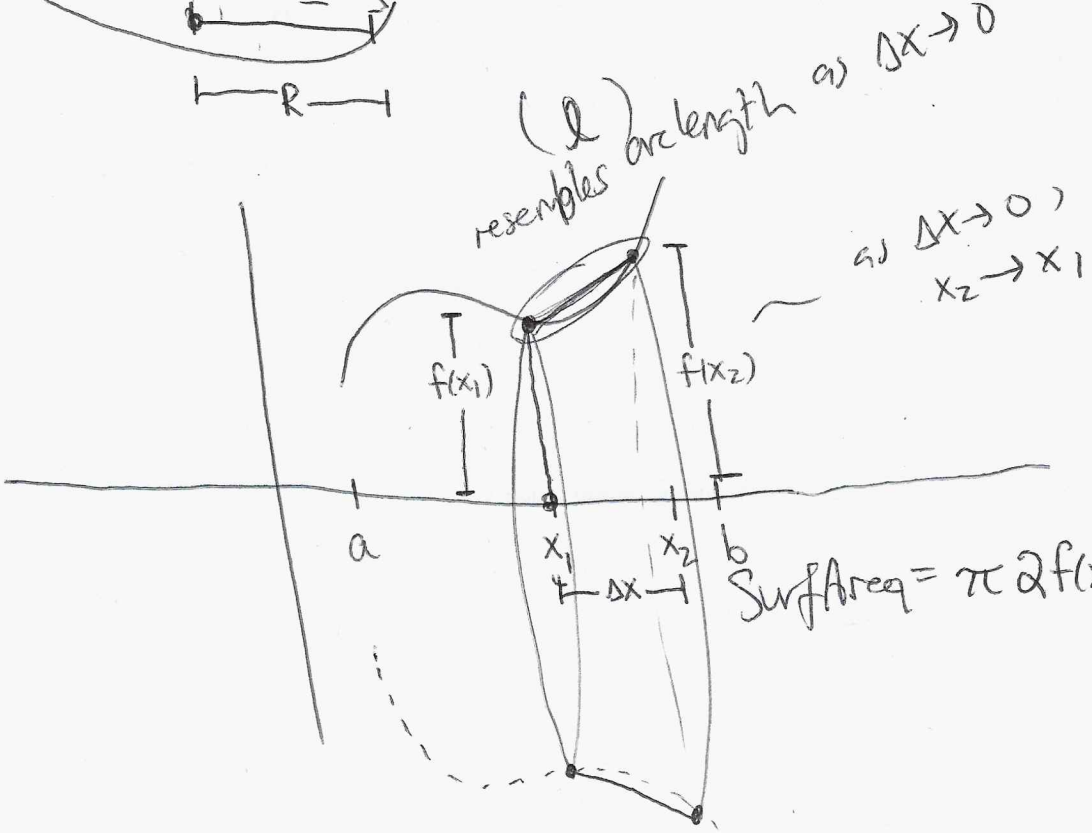


$SurfArea = \pi r s$

truncate Cone:



$SurfArea = \pi((R+r)l$



$SurfAreq = \pi 2 f(x) \sqrt{1+f'(x)^2}$

rotate x-axis:

$$\text{SurfArea} = 2\pi \int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$

rotate y-axis

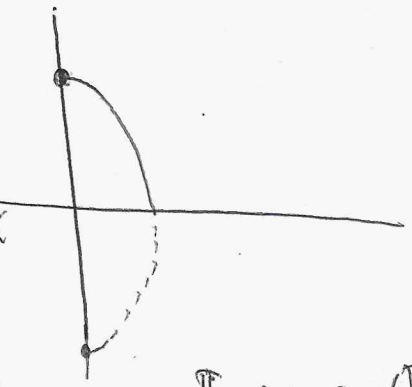
$$\text{SurfArea} = 2\pi \int_c^d g(y)\sqrt{1+g'(y)^2}\,dy$$

Ex: Graph $f(x) = \cos(5x)$ on $\left[0, \frac{\pi}{10}\right]$

$f'(x) = -5\sin(5x)$

rotate about x-axis.

$$\text{SurfArea} = 2\pi \int_0^{\pi/10} \cos(5x)\sqrt{1+25\sin^2(5x)}\,dx$$



$$= \frac{2\pi}{5}\int_0^1 \sqrt{1+25u^2}\,du$$

can do $u = \sin(5x)$

$\frac{1}{5}du = \cos(5x)\,dx$

$x = \frac{\pi}{10} \to u = \sin\left(\frac{\pi}{2}\right) = 1$

a mess ≈ 3.5

involves $\sinh^{-1}$ ← no thanks!

Ex: Curve $y = 7\sqrt{x+1} = 7(x+1)^{1/2}$ on $[2,3]$.

$y' = \frac{7}{2}(x+1)^{-1/2}$ ← (4)

$(a^b)^c = a^{bc}$

$((x+1)^{-1/2})^2 = (x+1)^{-1}$

$$\text{Surf Area} = 2\pi \int_2^3 \left(7\sqrt{x+1}\right)\sqrt{1+\frac{49}{4(x+1)}}\,dx$$

$$= 14\pi \int_2^3 \sqrt{x+1+\frac{49}{4}}\,dx$$

$$= 14\pi \int_2^3 \sqrt{x+\frac{53}{4}}\,dx \qquad u = x+\frac{53}{4}$$

$du = dx$

$x=2 \to u = \frac{61}{4}$

$x=3 \to u = \frac{65}{4}$

$$= 14\pi \int_{61/4}^{65/4} u^{1/2}\,du$$

$$= 14\pi \left.\frac{u^{3/2}}{3/2}\right|_{61/4}^{65/4} = \frac{28\pi}{3}\left[\left(\frac{65}{4}\right)^{3/2} - \left(\frac{61}{4}\right)^{3/2}\right]$$

$\sqrt{a}\sqrt{b} = \sqrt{ab}$ iff $a,b \geq 0$

why require pos?

$a=b=-1$

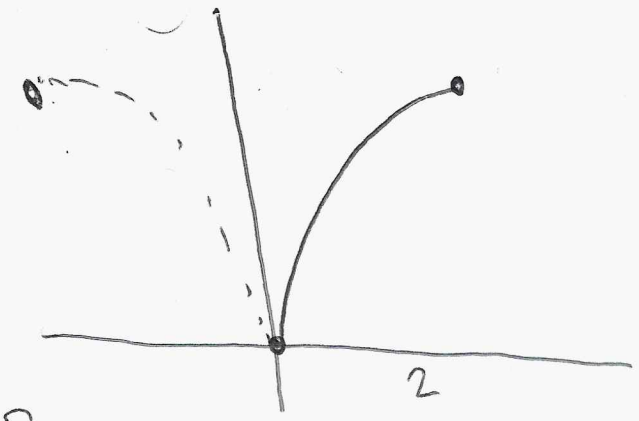
$\sqrt{1} = \sqrt{ab} \neq \sqrt{a}\sqrt{b}$

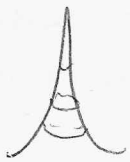
$= i \cdot i$

$= -1$

Ex: $x = \frac{y^3}{6}$, $0 \le y \le 2$, rotate y-axis

$\rightarrow x'(y) = \frac{y^2}{2}$





$$\text{Surf Area} = 2\pi \int_0^2 \frac{y^3}{6} \sqrt{1 + \left(\frac{y^2}{2}\right)^2}\, dy$$

$u = 1 + \frac{y^4}{4}$

$du = y^3\, dy$

$$= 2\pi \int_0^2 \frac{y^3}{6} \sqrt{1 + \frac{y^4}{4}}\, dy$$

$y = 0 \rightarrow u = 1$

$y = 2 \rightarrow u = 5$

$$= \frac{2\pi}{6} \int_1^5 u^{1/2}\, du$$

$$= \frac{\pi}{3} \left. \frac{u^{3/2}}{3/2} \right|_1^5 = \frac{2\pi}{9}\left[5^{3/2} - 1\right]$$