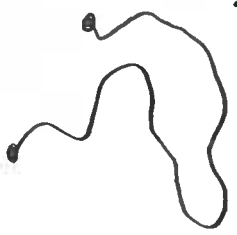
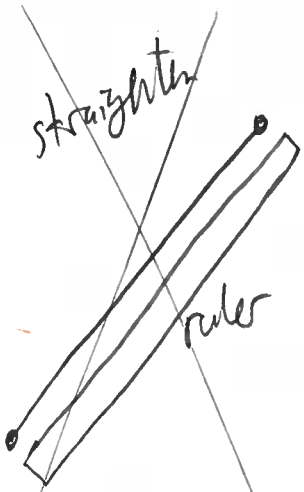
# Arc Length

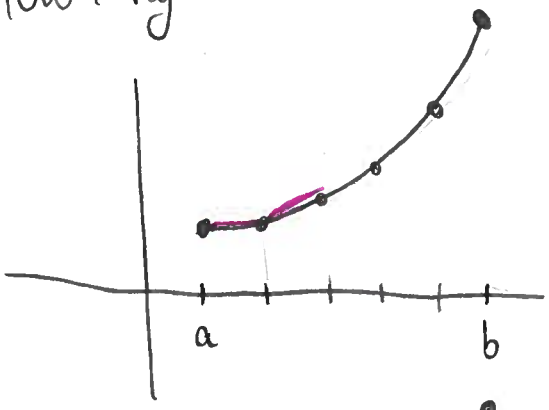


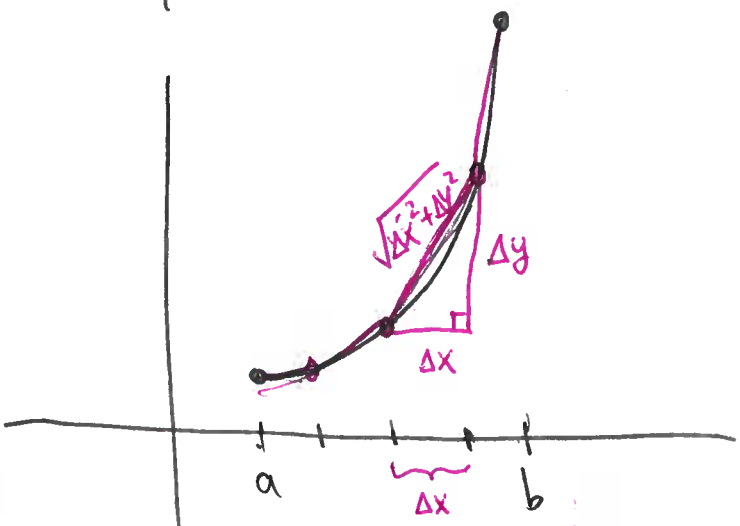

~length?




ruler

How long is a curve?




Crude estimate of length $= \sum \sqrt{\Delta x^2 + \Delta y^2}$

algebra

$= \sum \Delta x \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$

In limit: (as $\Delta x \to 0$)

$$\text{Arclength} = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

$dx$

Ex: Find arc length of $f(x) = 2x^{3/2}$ on $[0,1]$.

Solu: $f'(x) = 2\left(\frac{3}{2}\right)x^{1/2} = 3\sqrt{x}$

$$(f'(x))^2 = 9x$$

Therefore,

$$\text{Arclength} = \int_{x=0}^{x=1} \sqrt{1+9x}\, dx$$

$u = 1+9x$

$\frac{1}{9}du = dx$

if $x=0 \to u = 1+0 = 1$

$x=1 \to u = 1+9\cdot 1 = 10$

$$= \frac{1}{9}\int_{u=1}^{u=10} u^{1/2}\, du$$

$$= \frac{1}{9}\left[\frac{u^{3/2}}{3/2}\right]_1^{10}$$

$$= \frac{2}{27}\left[10^{3/2} - 1^{3/2}\right]$$

$$\approx 2.27$$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}$

Like #1 in OHW6 Find arclength of curve $y = \ln(\sin(x))$ between $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

$\frac{d}{dx}\ln(x) = \frac{1}{x}$

Soln: $y' = \frac{1}{\sin(x)}\cos(x)$

$\frac{dy}{dx} = \frac{d}{dx}\ln(\sin(x))$

$= \frac{d(\sin(x))}{dx}\left\{\frac{d}{d(\sin x)}\ln(\sin x)\right\}$

$= \cos(x)\frac{1}{\sin(x)}$

$(y')^2 = \left(\frac{\cos(x)}{\sin(x)}\right)^2 = \frac{\cos^2(x)}{\sin^2(x)}$

$= \cot^2(x)$

Pyth identity

$\cos^2 x + \sin^2 x = 1$

↓ div by $\sin^2 x$

$\cot^2(x) + 1 = \csc^2(x)$

$\int \cot(x)\cos(x)$

$= \int \frac{\cos^2 x}{\sin x}$

$u = \sin x$

$du = \cos^2 x\, dx$

$\frac{d}{dx}\frac{1}{\cos x} = \frac{+\sin(x)}{\cos^2(x)}$

$\text{Arclength} = \int_{\pi/6}^{\pi/4}\sqrt{1+\cot^2(x)}\,dx$

$= \int_{\pi/6}^{\pi/4}\sqrt{\csc^2(x)}\,dx$

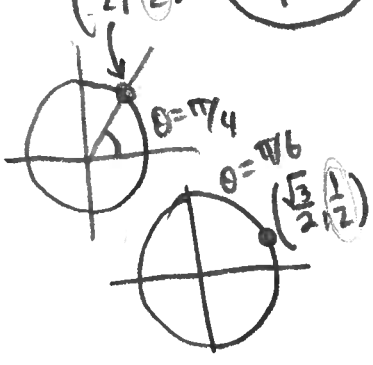
$= \int_{\pi/6}^{\pi/4}\csc(x)\,dx = \int_{\pi/6}^{\pi/4}\frac{1}{\sin x}\,dx$

look up → $= -\ln(\csc(x)+\cot(x))\Big|_{\pi/6}^{\pi/4}$

$= -\ln(\csc(\tfrac{\pi}{4})+\cot(\tfrac{\pi}{4})) + \ln(\csc(\tfrac{\pi}{6})+\cot(\tfrac{\pi}{6}))$

$= -\ln\left(\frac{2}{\sqrt{2}}+1\right) + \ln\left(2+\frac{\sqrt{3}/2}{1/2}\right)$

$= \ln(2+\sqrt{3}) - \ln\left(\frac{2}{\sqrt{2}}+1\right) \approx 0.44$




Relevant to OHW6

$\int \sec(x)\,dx$

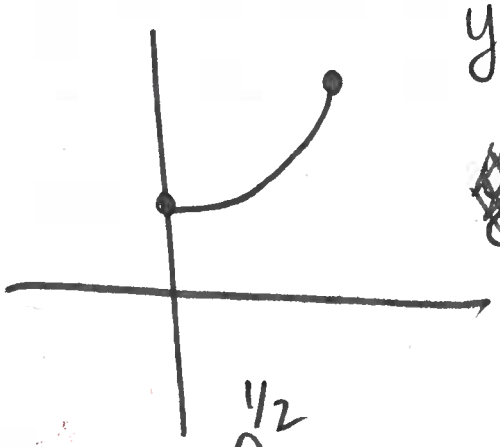
$= \ln(\tan(x)+\sec(x)) + C$

Ex: Find arclength of $y = \frac{e^{3x}}{5}$ on $[0, 1/2]$.

$(e^a)^b = e^{a \cdot b}$

$(e^{3x})^2 =$

Soln:



$y' = \frac{3}{5} e^{3x}$

$(y')^2 = \left(\frac{9}{25}\right) e^{6x}$

$\sqrt{1+x^2}$

$$\text{Arclength} = \int_0^{1/2} \sqrt{1 + \frac{9}{25} e^{6x}}\, dx$$

how to $\int$?

don't want to go there!

computer $= 0.87$