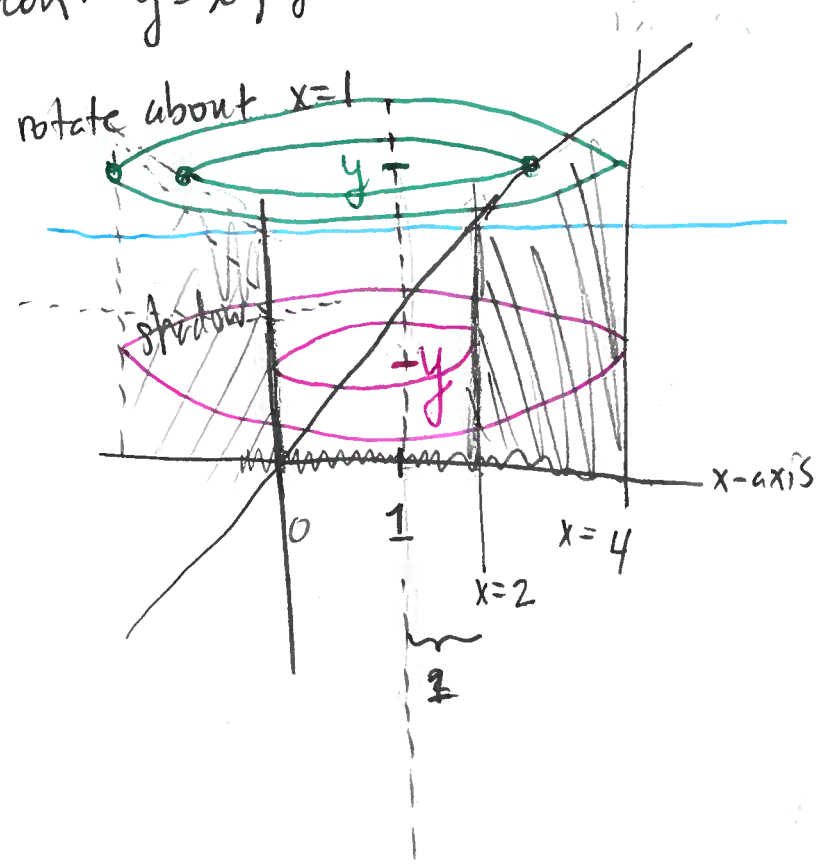
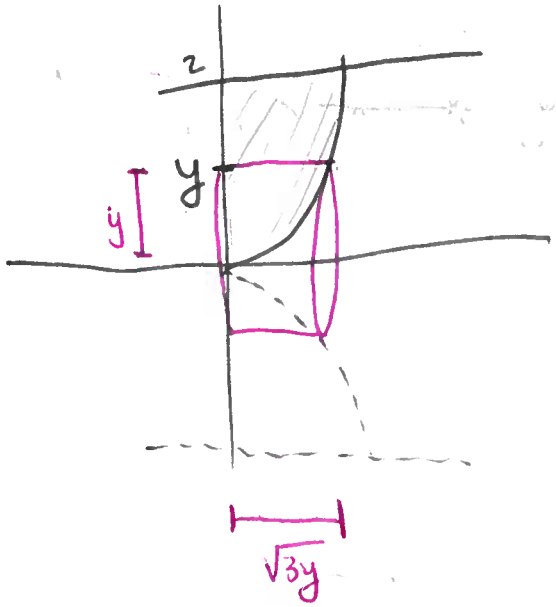
Region: $y=x$, $y=0$, $x=2$, $x=4$



Ex: Region bdd by $x=\sqrt{3y}$, $x=0$, and $y=2$ rotated about x-axis.

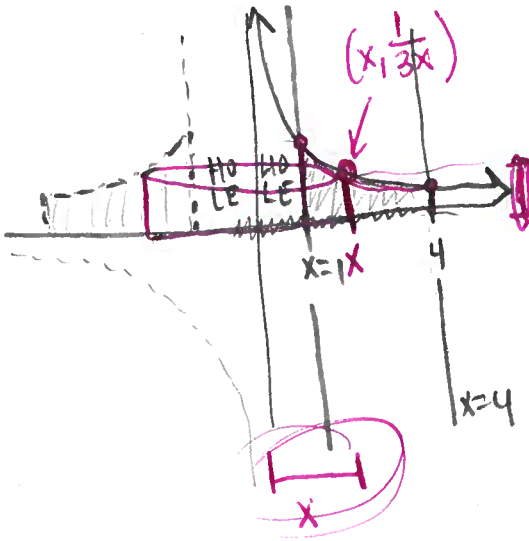



$$Vol = 2\pi \int_0^2 y\sqrt{3y}\,dy$$

$$= 2\sqrt{3}\pi \int_0^2 y^{3/2}\,dy$$

$$= 2\sqrt{3}\pi \cdot \frac{2}{5}\, 2^{5/2}$$

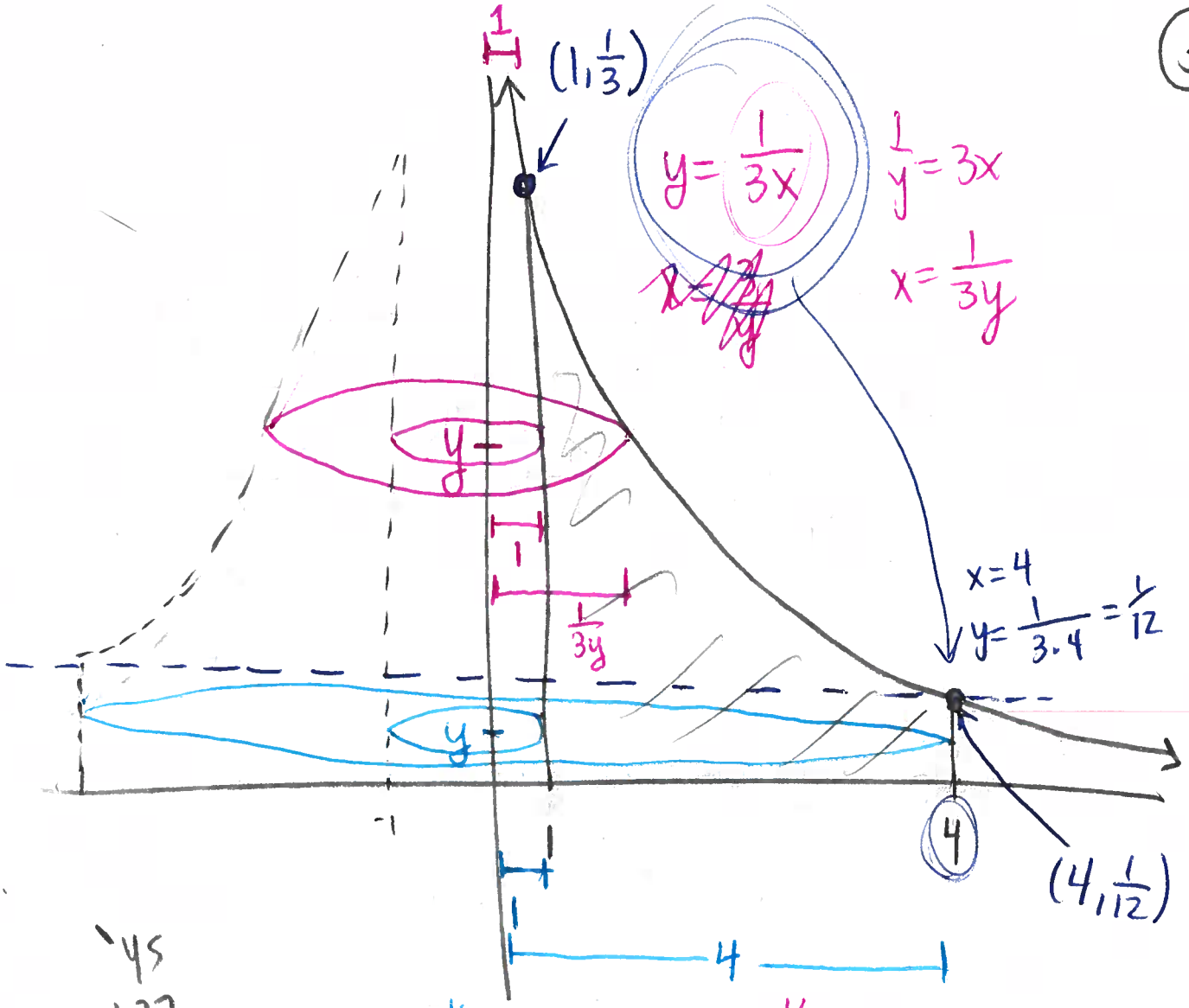
Ex: Region bdd by $y=\frac{1}{3x}$, $x=1$, $x=4$, $y=0$ rotate y-axis.




$$Vol = 2\pi \int_1^4 x \cdot \frac{1}{3x}\,dx$$

$$= 2\pi \int_1^4 \frac{1}{3}\,dx$$

$$= \frac{2\pi}{3}\left[x\right]_1^4 = 2\pi$$




$$\text{Vol} = \pi \int_0^{1/12} 4^2 - 1^2\,dy + \pi \int_{1/12}^{1/3} \left(\frac{1}{3y}\right)^2 - 1^2\,dy$$

$$= \pi \int_0^{1/12} 15\,dy + \pi \int_{1/12}^{1/3} \frac{1}{9}y^{-2} - 1\,dy$$

$$= 15\pi\left[\frac{1}{12}\right] + \pi\left[-\frac{1}{9y} - y\right]_{1/12}^{1/3}$$

$$= \frac{15\pi}{12} + \pi\left\{\left[-\frac{3}{9} - \frac{1}{3}\right] - \left[-\frac{12}{9} - \frac{1}{12}\right]\right\}$$

$$= \frac{15\pi}{12} + \pi\left\{-\frac{6}{9} - \left(-\frac{51}{36}\right)\right\}$$

$$= \frac{15\pi}{12} + \frac{27\pi}{36} = \frac{72\pi}{36} = 2\pi \checkmark$$

$4\,11$ $51$ $-24$ $27$

$45$ $+27$ $72$

$\frac{6}{9} = \frac{24}{36}$

$\frac{-24+51}{36}$

$\frac{15}{12} = \frac{45}{36}$

$\frac{1}{1/3} = 3$

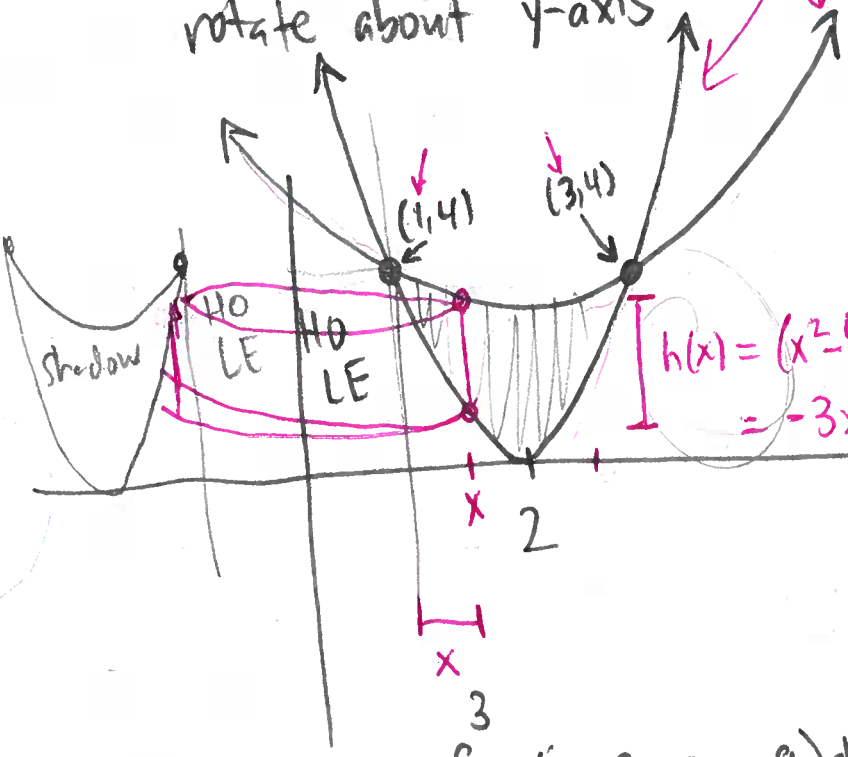
$\frac{12}{9} = \frac{48}{36}$

$\frac{3}{12} = \frac{9}{36}$

**Ex:** region bdd by $y = 4(x-2)^2$ and $y = x^2 - 4x + 7$ rotate about y-axis

$= 4(x^2 - 4x + 4)$

$= 4x^2 - 16x + 16$




2 pts

$h(x) = (x^2 - 4x + 7) - (4x^2 - 16x + 16)$

$= -3x^2 + 12x - 9$

$$\text{Vol} = 2\pi \int_1^3 x\underbrace{(-3x^2 + 12x - 9)}_{-3x^3 + 12x^2 - 9x}\,dx$$

$$= 2\pi \left[ -\frac{3}{4}x^4 + 4x^3 - \frac{9}{2}x^2 \right]_1^3$$

$$= 2\pi \left[ \frac{27}{4} - \left(-\frac{5}{4}\right) \right]$$

$$= 2\pi \left[ \frac{32}{4} \right] = 2\pi \cdot 8 = 16\pi$$