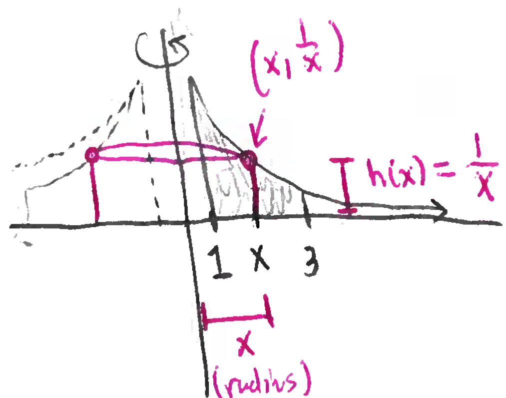
Ex: region bdd above by $\frac{1}{x}$,
bdd below by x-axis
above $[1,3]$
rotate about y-axis

Soln:



$$\text{Volume} = 2\pi \int_1^3 x \cdot \frac{1}{x}\, dx$$

$$= 2\pi \int_1^3 1\, dx$$

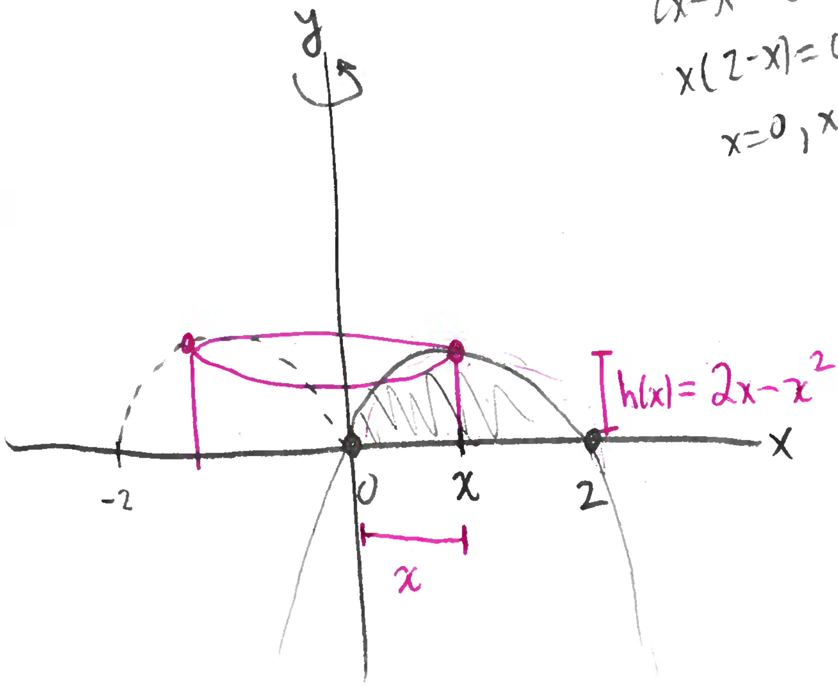
$$= 2\pi x \Big|_1^3 = 2\pi\left[(3) - 1\right]$$

$$= 4\pi$$

(2)

(2) region bdd above by $2x-x^2$, x-axis [0,2] on rotate y-axis

$2x-x^2=0$
$x(2-x)=0$
$x=0,\ x=2$



$$Vol = 2\pi\int_0^2 x\cdot(2x-x^2)\,dx$$

$$= 2\pi\int_0^2 2x^2-x^3\,dx$$

$$2\cdot16 \quad \frac{4}{64}$$

$$= 2\pi\left[\frac{2}{3}x^3-\frac{x^4}{4}\right|_0^2$$

$$\cdot 16 \quad \frac{3}{48}$$

$$= 2\pi\left[\left(\frac{16}{3}-\frac{16}{4}\right)-0\right]$$

$$= 2\pi\left[\frac{64}{12}-\frac{48}{12}\right]$$

$$= 2\pi\left[\frac{16}{12}\right] = \frac{8\pi}{3}$$

$$\frac{8}{6} \quad \frac{4}{3}$$

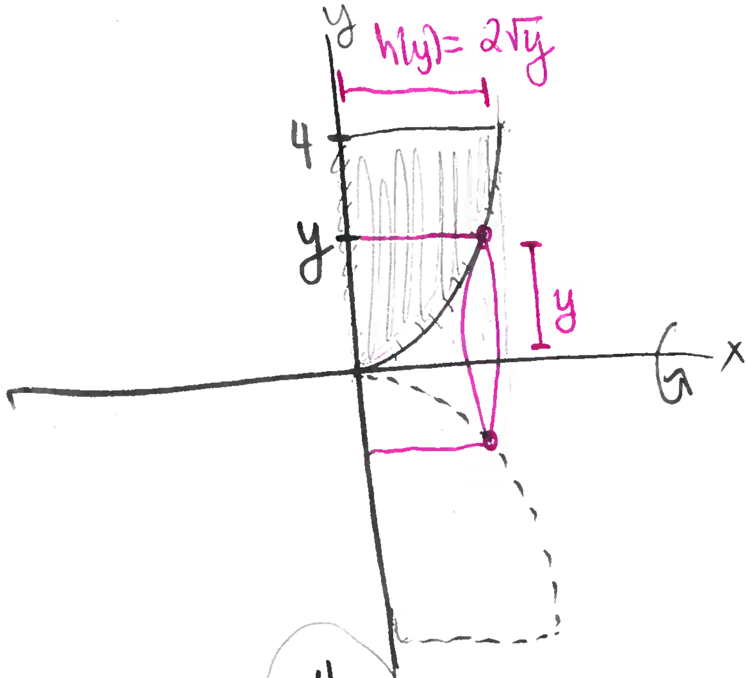
Ex: Region bdd on right by $x = 2\sqrt{y}$, on left by y-axis for $y \in [0,4]$

Rotate about x-axis.

③

$\frac{x}{2} = \sqrt{y}$

$y = \frac{x^2}{4}$



$a^{\frac{b}{c}} = \sqrt[c]{a^b}$

$y\sqrt{y} = y y^{1/2} = y^{1+\frac{1}{2}}$

$$\text{Vol} = 2\pi \int_0^4 y \cdot 2\sqrt{y}\, dy$$

$4^{5/2} = \sqrt{4^5}$

$\frac{\frac{a}{b}}{\frac{c}{a}} = \frac{d}{c} \cdot \frac{a}{b}$

$$= 4\pi \int_0^4 y^{3/2}\, dy$$

$= 2^5$

$$= 4\pi \left. \frac{y^{5/2}}{5/2} \right|_0^4 = \frac{8\pi}{5}\left[4^{5/2} - 0^{5/2} \right]$$

$32 \times 8 = 256$

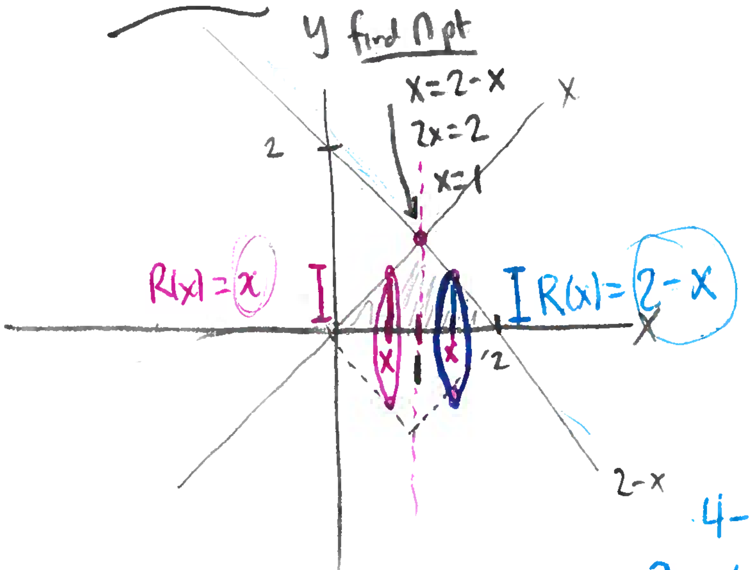
$$= \frac{8\pi}{5}(32)$$

$$= \frac{256\pi}{5}$$

Ex: Which method is best?

Consider region bdd by $y=x$, $y=2-x$, x-axis rotated about x-axis.

washer



$4-4x+x^2$

$$\text{Vol} = \pi\int_0^1 x^2\,dx + \pi\int_1^2 (2-x)^2\,dx$$

$$= \pi\left(\frac{1}{3}\right) + \pi\left[4x - 2x^2 + \frac{1}{3}x^3\right|_1^2$$

$$= \frac{\pi}{3} + \pi\left[\left(8 - 8 + \frac{8}{3}\right) - \left(4 - 2 + \frac{1}{3}\right)\right]$$

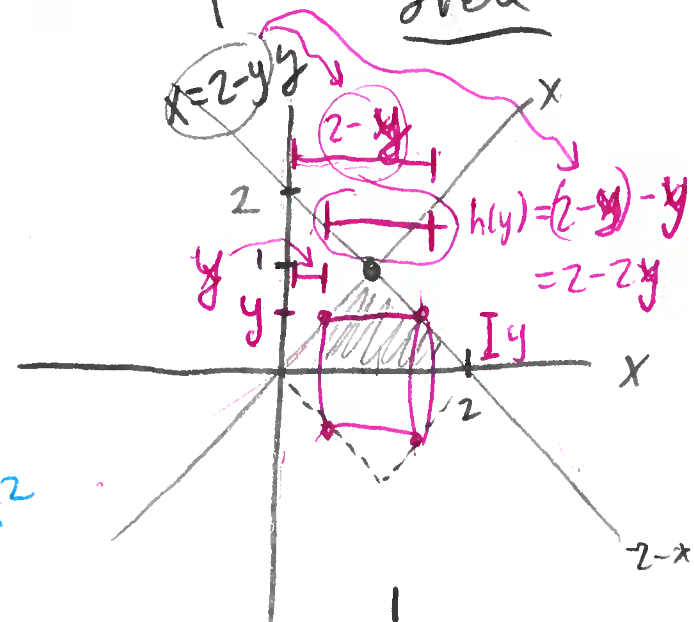
$2+\frac{1}{3}=$

$$= \frac{\pi}{3} + \cancel{8}\,\pi\left[\frac{8}{3} - \frac{7}{3}\right]$$

$\cancel{+\frac{1}{3}\pi}$

$$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

shell



$$\text{Vol} = 2\pi\int_0^1 y\cdot(2-2y)\,dy$$

$$= 2\pi\int_0^1 2y - 2y^2\,dy$$

$$= 2\pi\left[1 - \frac{2}{3}\right]$$

$$= 2\pi\left(\frac{1}{3}\right)$$

$$= \frac{2\pi}{3}$$

Both methods here have upsides & downsides. "Best" is subjective here.

$x=2$

$y''=-2$

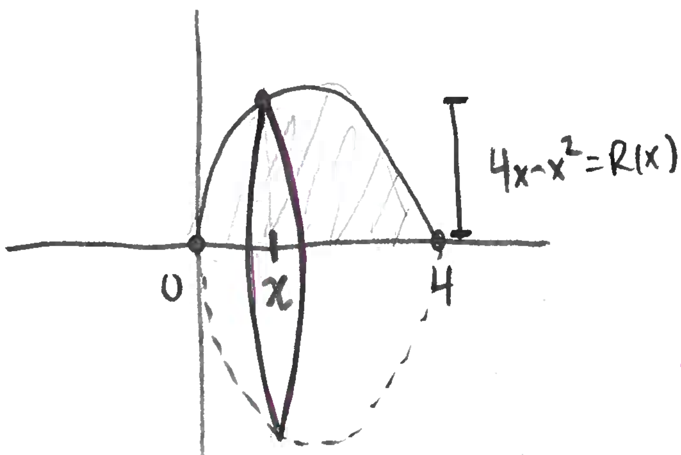
Ex: Which method best?

Region bdd by $y=4x-x^2$ and x-axis.

Rotate about x-axis.

need $x=$ fnct of y

washer



$$\text{Vol}=\pi\int_0^4 (4x-x^2)^2\,dx$$

$$=\pi\int_0^4 16x^2-8x^3+x^4\,dx$$

$$=\pi\left[\frac{16}{3}x^3-2x^4+\frac{x^5}{5}\right]_0^4$$

$$=\pi\left[\left(\frac{16\cdot 4^3}{3}-2\cdot 4^4+\frac{4^5}{5}\right)-0\right]$$

≈ 107.23

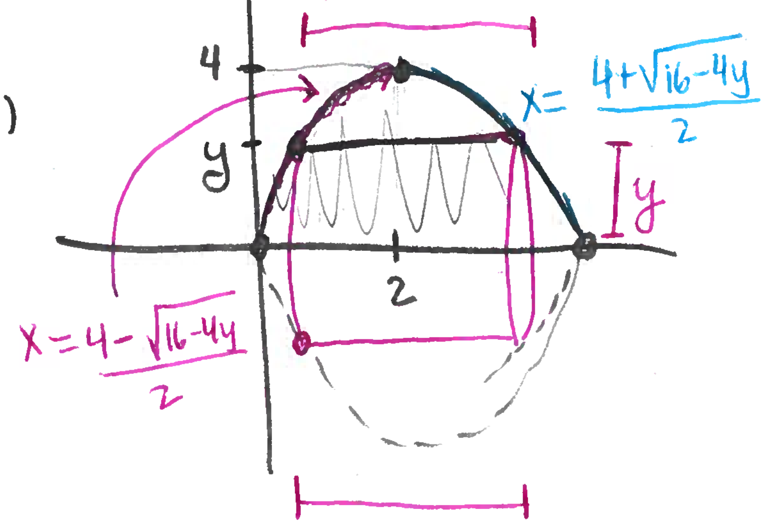
CLEARLY~

washer is best here!

shell

$$x^2-4x+y=0$$

$$x=\frac{4\pm\sqrt{16-4y}}{2}$$



$$R(y)=\frac{4+\sqrt{16-4y}}{2}-\frac{4-\sqrt{16-4y}}{2}$$

$$=\sqrt{16-4y}$$

$$\text{Vol}=2\pi\int_0^4 y\sqrt{16-4y}\,dy \qquad u=16-4y$$

$$\frac{du}{-4}=dy$$

$$y=-4-\frac{u}{4}$$

?

$$=\frac{2\pi}{-4}\int_{16}^{0}\left(-4-\frac{u}{4}\right)\sqrt{u}\,du$$

$u^{1/2}$

$$=(-)\frac{\pi}{2}\int_{16}^{0}-4u^{1/2}-\frac{u^{3/2}}{4}\,du$$

$$=\frac{\pi}{2}\left[-\frac{4u^{3/2}}{3/2}-\frac{u^{5/2}}{4\cdot(5/2)}\right]_0^{16}=\frac{\pi}{2}\left[-\frac{8}{3}16^{3/2}-\frac{16^{5/2}}{10}\right]$$

somehow ≈ 107.23