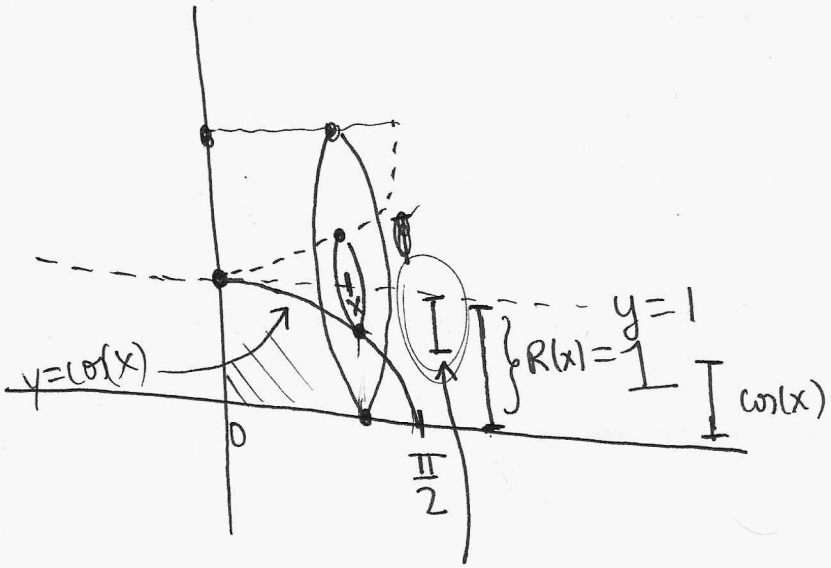
Like Problem 7 OHW4



$$r(x) = 1 - \cos(x)$$

$$\text{Vol} = \pi \int_0^{\pi/2} \underbrace{1^2}_{R(x)^2} - \underbrace{(1-\cos(x))^2}_{r(x)^2}\, dx$$

$$= \pi \int_0^{\pi/2} 1 - \left[1 - 2\cos(x) + \cos^2(x)\right] dx$$

$$= \pi \int_0^{\pi/2} 2\cos(x) - \cos^2(x)\, dx$$

Q: How to $\int_0 \cos^2(x)$?

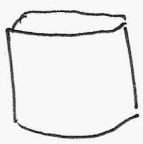
$$\int \cos^2(x)\, dx$$

$$= \int \frac{1+\cos(2x)}{2}\, dx$$

$$= \frac{1}{2}\int 1\, dx + \frac{1}{2}\int \cos(2x)\, dx$$

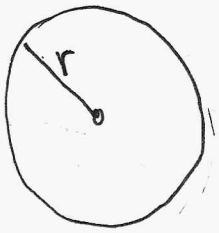
can do $u = 2x$ to integrate!!

Shell Method for volume

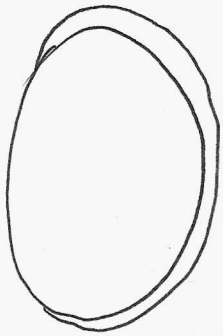


cylindrical shell

based on surface area:



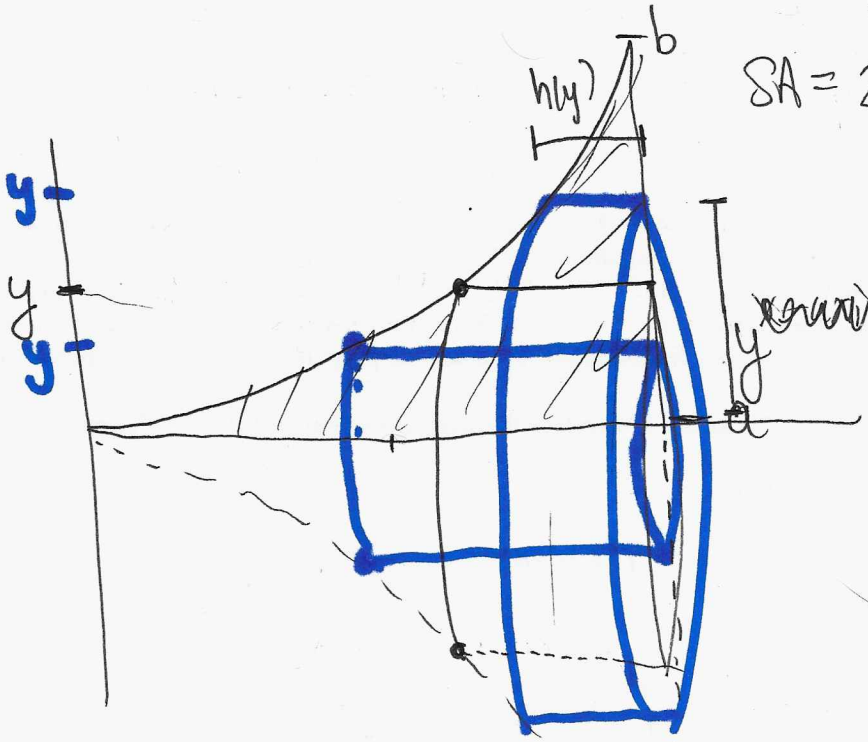
Circumference $= 2\pi r$



"shell"

surface area of outer part of ring

$SA = 2\pi r h$



$$Vol = 2\pi \int_a^b h(y)\, y\, dy$$

Rule:

	washer	shell
x-axis rotation →	dx	dy
y-axis rotation →	dy	dx

Ex (from 1 February):

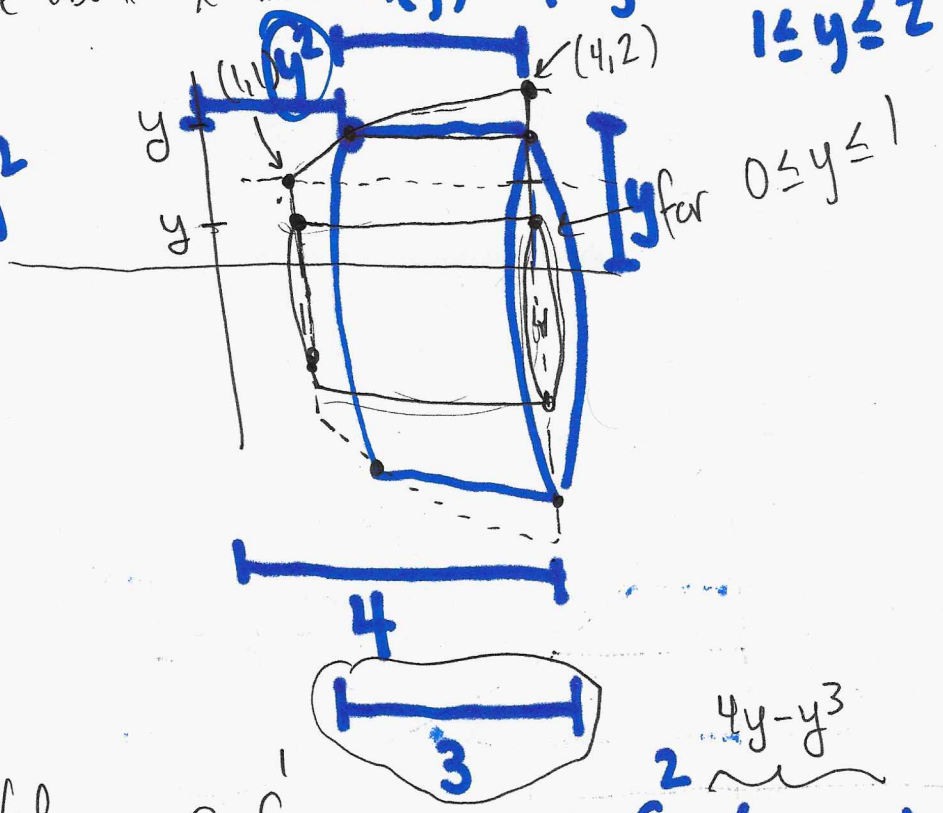
bdd by $y=\sqrt{x}$ above

x-axis below on $[1,4]$

rotate about x-axis

$x=y^2$

$h(y)=4-y^2$ ← valid for $1\le y\le 2$



$$\text{Volume} = 2\pi\int_0^1 y\cdot 3\,dy + 2\pi\int_1^2 \underbrace{y(4-y^2)}_{4y-y^3}\,dy$$

0 ↑ radius, height ↑

$$= 2\pi\left[\frac{3y^2}{2}\Big|_0^1 + 2\pi\left[2y^2-\frac{y^4}{4}\Big|_1^2\right.\right.$$

$8=\frac{32}{4}$, $2=\frac{8}{4}$

$$= 2\pi\left[\frac{3}{2}-0\right] + 2\pi\left[\left(8-\frac{16}{4}\right)-\left(2-\frac{1}{4}\right)\right]$$

$$= 3\pi + 2\pi\left[\frac{16}{4}-\frac{7}{4}\right] = 3\pi + 2\pi\left[\frac{9}{4}\right]$$

$$= \frac{12}{4}\pi+\frac{18}{4}\pi = \frac{30}{4}\pi = \frac{15\pi}{2}$$