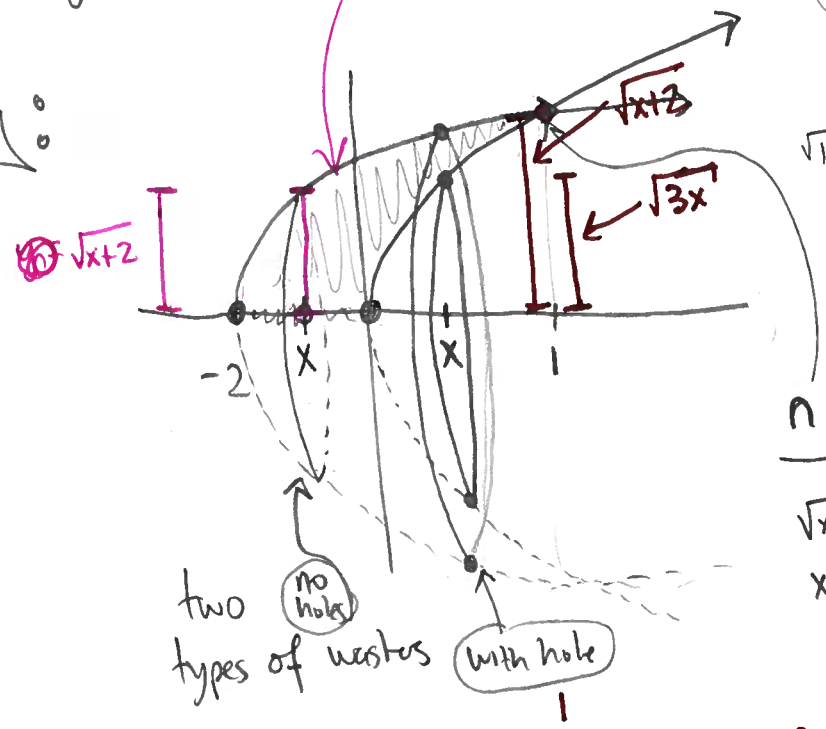




Ex: Region bdd by  $y = \sqrt{x+2}$  and  $y = \sqrt{3x}$  and  $y=0$  revolved around x-axis.

Soln:



$\sqrt{3} > 1$   
 $1 < 2 < 3 < 4$   
 $\sqrt{1} < \sqrt{2} < \sqrt{3} < \sqrt{4}$   
 $1 < \sqrt{3} < 2$

two types of washers  
 (no hole) with hole

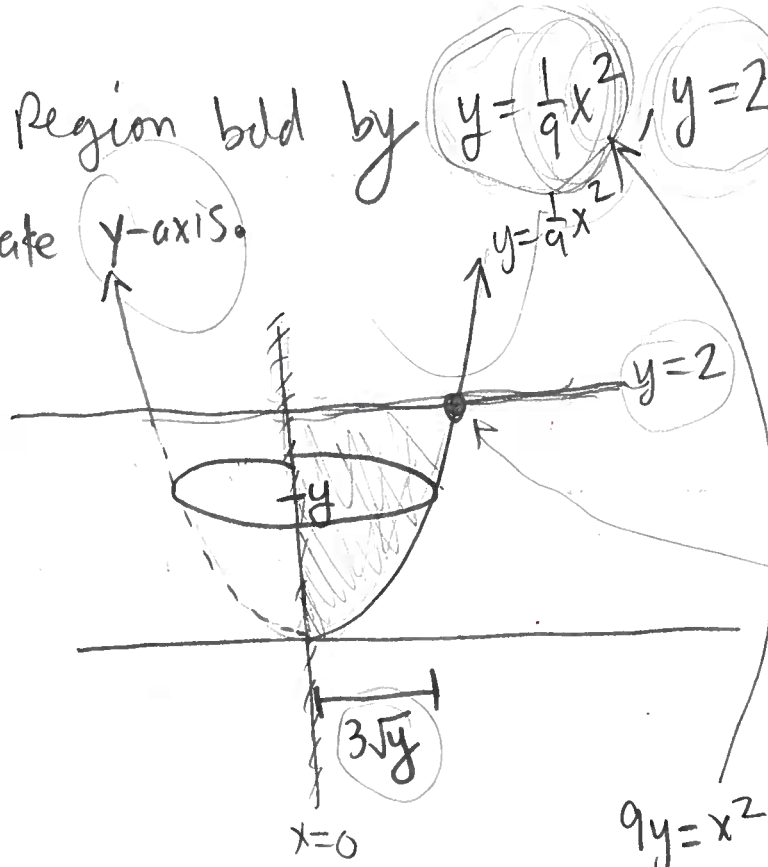
n pt  
 $\sqrt{x+2} = \sqrt{3x}$   
 $x+2 = 3x$   
 $2 = 2x$

$x=1$

$$\begin{aligned} \text{Total Vol} &= \pi \int_{-2}^0 (\sqrt{x+2})^2 dx + \pi \int_0^1 (\sqrt{x+2})^2 - (\sqrt{3x})^2 dx \\ &= \pi \int_{-2}^0 x+2 dx + \pi \int_0^1 x+2 - 3x dx \\ &= \pi \left[ 0 - \left( \frac{(-2)^2}{2} + 2(-2) \right) \right] + \pi \left[ -1^2 + 2 - [0] \right] \\ &= \pi [2] + \pi \\ &= 3\pi \end{aligned}$$

Ex: Region bdd by  $y = \frac{1}{9}x^2$ ,  $y = 2$ ,  $x = 0$ ,  $x > 0$

rotate  $y$ -axis.



$$\begin{aligned} \frac{1}{9}x^2 &= 2 \\ x^2 &= 18 \\ x &= \pm\sqrt{18} = \pm 3\sqrt{2} \end{aligned}$$

$\pi r^2$

$$\begin{aligned} x &= 3\sqrt{y} \\ x &= 3\sqrt{2} \end{aligned}$$

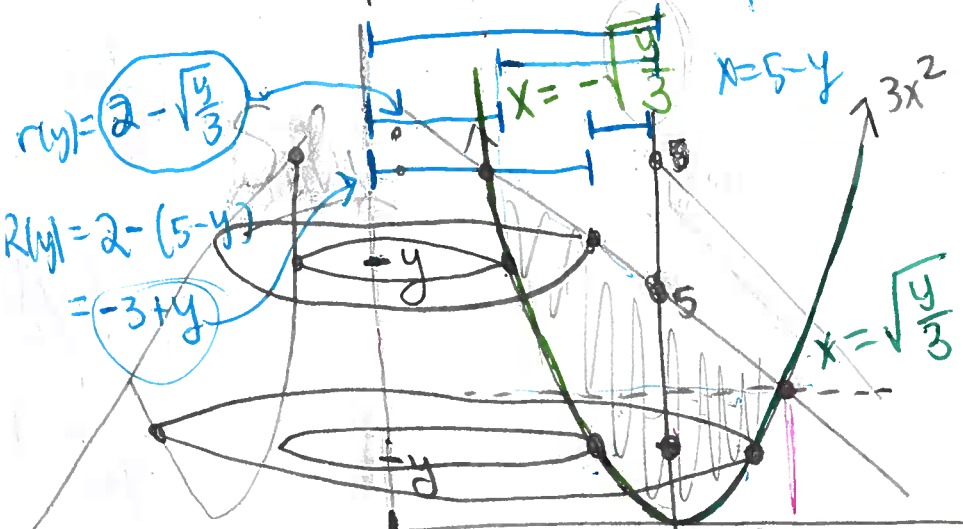
$$\begin{aligned} 9y &= x^2 \\ x &= \pm\sqrt{9y} \\ x &= 3\sqrt{y} \end{aligned}$$

$$\begin{aligned} \text{Vol} &= \int_0^2 \pi (3\sqrt{y})^2 dy \\ &= 9\pi \int_0^2 y dy \\ &= 9\pi \left[ \frac{y^2}{2} - 0 \right] \\ &= 18\pi \end{aligned}$$

Ex: Region bdd by  $y = 3x^2$

and  $x + y = 5$

rotated about line  $x = -2$ .



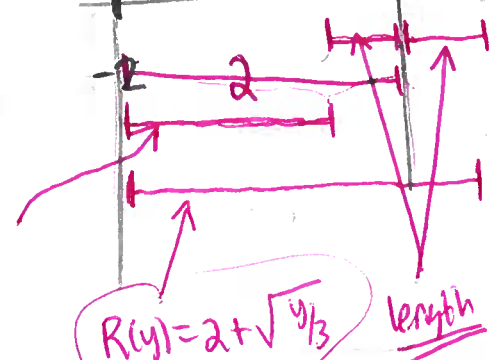
$y = 5 - x$

$3x^2 = 5 - x$

$3x^2 + x - 5 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-5)}}{6}$

$= \frac{-1 \pm \sqrt{61}}{6}$



$r(y) = 2 - \sqrt{\frac{y}{3}}$

$R(y) = 2 + \sqrt{\frac{y}{3}}$

length

$y = 3x^2$

$\frac{y}{3} = x^2$

$x = \pm \sqrt{\frac{y}{3}}$

3.865

$\sqrt{\frac{y}{3}}$

6.468

$$Vol = \pi \int_0^{3.865} (2 + \sqrt{\frac{y}{3}})^2 - (2 - \sqrt{\frac{y}{3}})^2 dy + \int_{3.865}^{6.468} (-3 + y)^2 - (2 - \sqrt{\frac{y}{3}})^2 dy$$

Come from desmos sketch