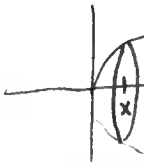
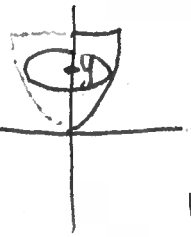
$$\text{Volume} = \pi \int_a^b R(x)^2 - r(x)^2 \, dx$$



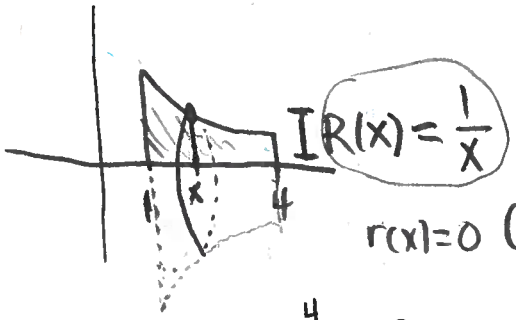
rotation about horizontal axis

$$\text{Volume} = \pi \int_c^d R(y)^2 - r(y)^2 \, dy$$



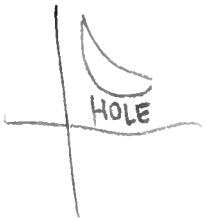
rotate about vertical axis

Ex: region defined by $y = \frac{1}{x}$, x-axis, above $[1,4]$
rotate about x-axis



$R(x) = \frac{1}{x}$

$r(x) = 0$ ("no hole")



$$\text{Vol} = \pi \int_1^4 \left(\frac{1}{x}\right)^2 dx = \pi \int_1^4 x^{-2} \, dx$$

$$= \pi \left[\frac{x^{-1}}{(-1)} \right]_1^4$$

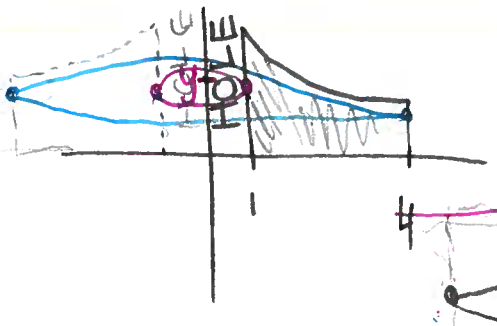
$$= \pi \left[-\frac{1}{4} - (-1) \right]$$

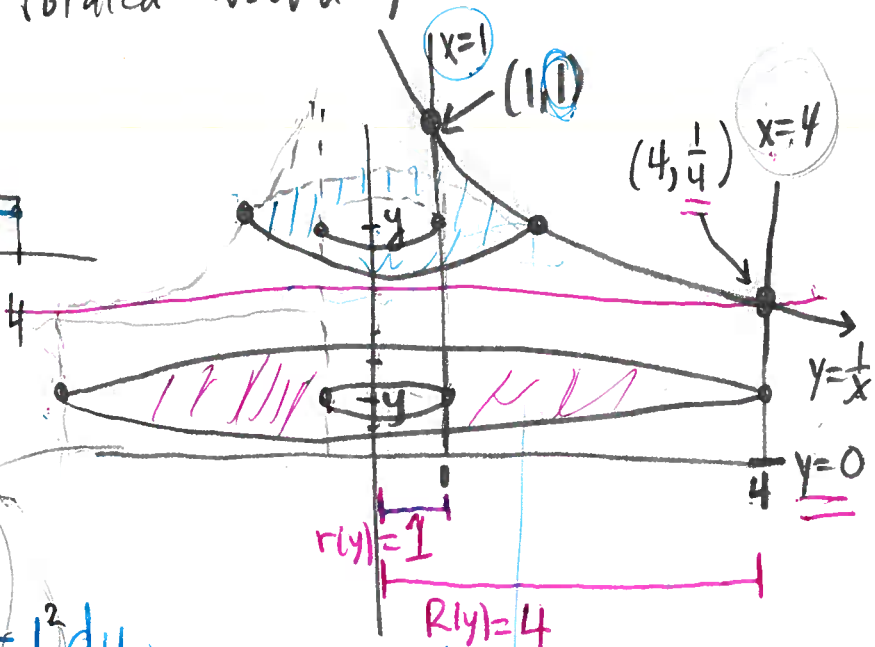
$$= \frac{3\pi}{4}$$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

Ex: Same region but rotated around y-axis





$$\text{Vol} = \pi\int_0^{1/4} \underbrace{4^2 - 1^2}_{15}\,dy + \pi\int_{1/4}^{1} \left(\frac{1}{y}\right)^2 - 1^2\,dy$$

$$= \pi\left[15y\Big|_0^{1/4}\right. + \pi\left[-\frac{1}{y} - y\Big|_{1/4}^{1}\right.$$

$$= \pi\left[\frac{15}{4} - 0\right] + \pi\left[(-1-1) - \left(-\frac{1}{1/4} - \frac{1}{4}\right)\right]$$

$$= \frac{15\pi}{4} + (-2\pi) + \frac{17}{4}\pi$$

$$= \frac{15\pi}{4} + \frac{9\pi}{4} = \frac{24\pi}{4} = 6\pi$$

$r(y) = 1$

$R(y) = \frac{1}{y}$

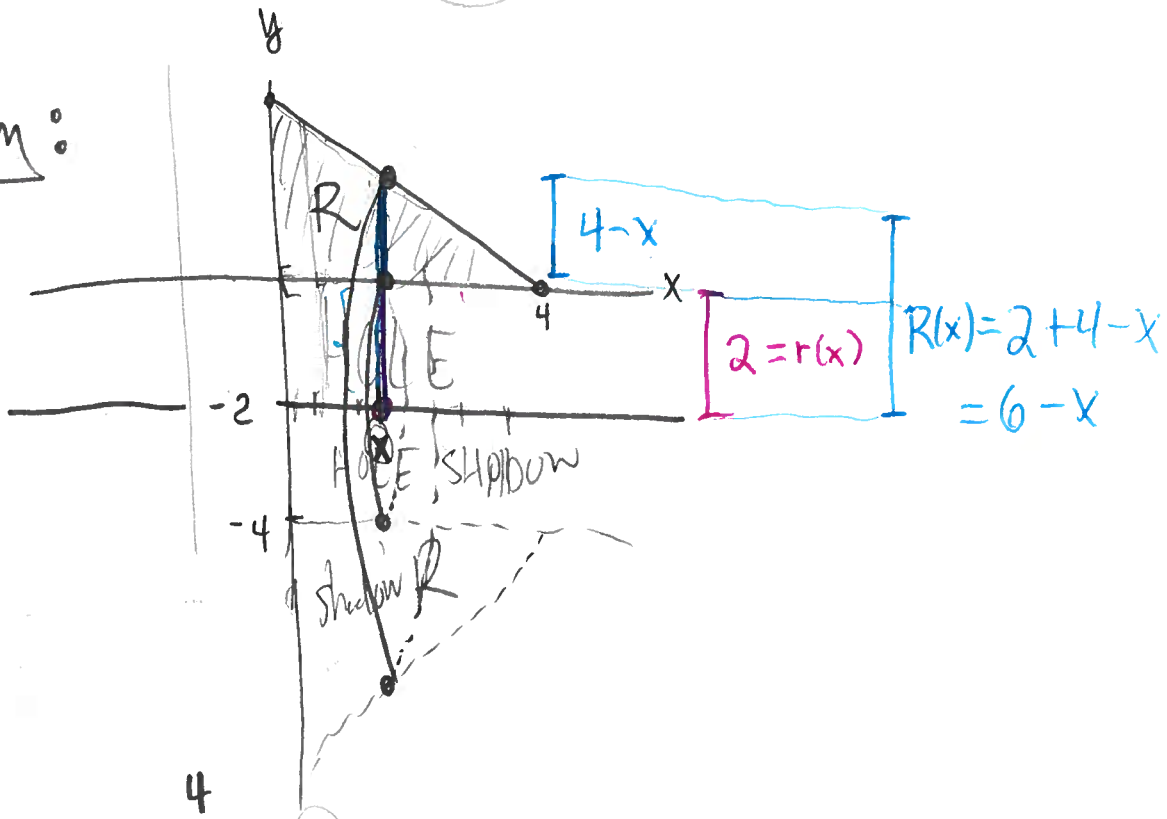
$y = \frac{1}{x}$

$x = \frac{1}{y}$

$-4 - \frac{1}{4} = -\frac{16}{4} - \frac{1}{4}$

Ex: $y=4-x$ above $[0,4]$ rotate about line $y=-2$

Soln:



$$\text{Vol} = \pi \int_0^4 (6-x)^2 - 2^2 \, dx$$

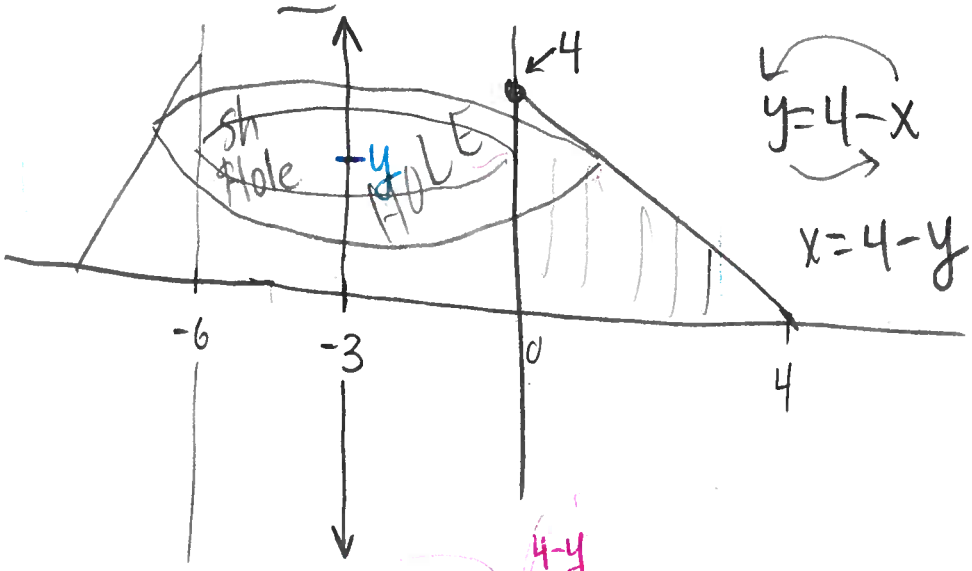
$$= \pi \int_0^4 \underbrace{36 - 12x + x^2 - 4}_{x^2 - 12x + 32} \, dx$$

$$= \pi \left[\frac{x^3}{3} - 6x^2 + 32x \right|_0^4$$

$$= \pi \left[\frac{64}{3} - 6 \cdot 16 + 32 \cdot 4 \right]$$

$$= \ldots$$

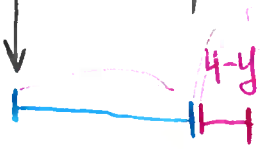
Ex: Same but rotate about $x=-3$



$y=4-2x$

$2x=4-y$

$x=2-\frac{y}{2}$



$r(y)=3$

$R(y)=3+(4-y)$

$=7-y$

$$Vol=\pi\int_0^4 (7-y)^2-3^2\,dy$$

$$=\pi\int_0^4 49-14y+y^2-9\,dy$$

$$y^2-14y+40$$

$$=\pi\left[\frac{y^3}{3}-7y^2+40y\right]_0^4$$

$$=\pi\left[\frac{64}{3}-7(16)+160\right]$$