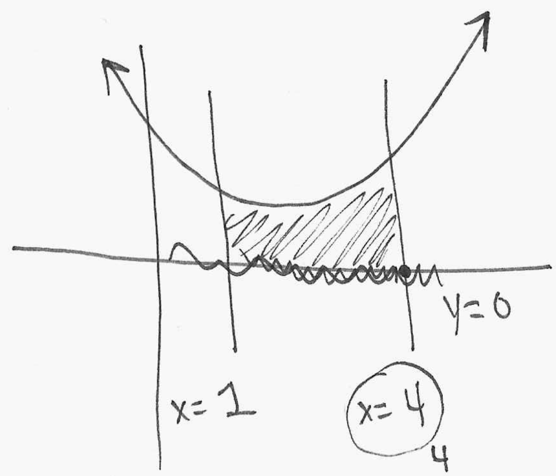
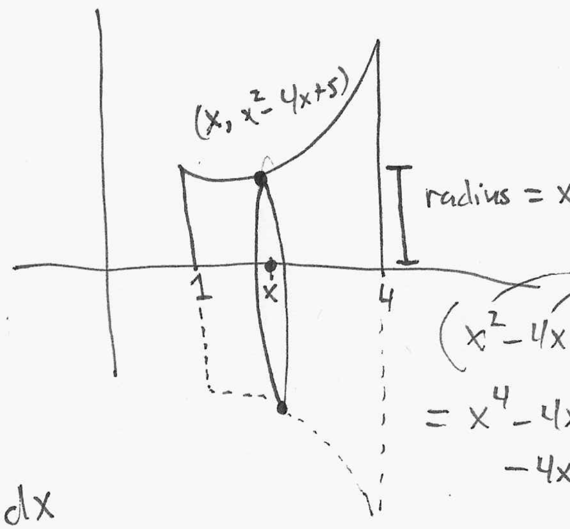
$$\pi \int_a^b R(x)^2 - r(x)^2 \, dx \qquad \pi \int_a^b R(y)^2 - r(y)^2 \, dy$$

Ex 2.7: Find vol of ~~surface~~ solid of rev bdd by graphs of $y = x^2 - 4x + 5$ and $x = 4$ and $y = 0$ and $x = 1$ rotate. about x-axis.



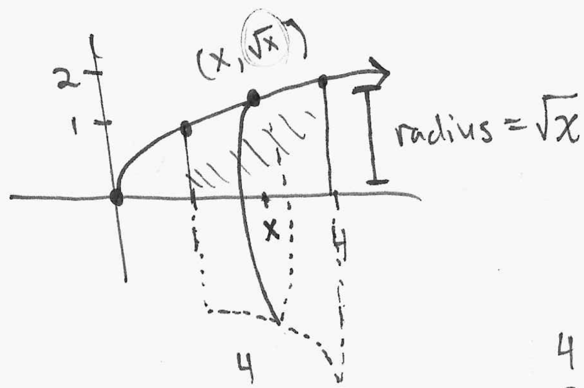


$$(x^2-4x+5)(x^2-4x+5)$$
$$= x^4 - 4x^3 + 5x^2$$
$$\quad - 4x^3 + 16x^2 - 20x$$
$$\quad + 5x^2 - 20x + 25$$
$$= x^4 - 8x^3 + 26x^2 - 40x + 25$$

$$\text{Volume} = \pi \int_1^4 (x^2-4x+5)^2 \, dx$$
$$= \pi \int_1^4 x^4 - 8x^3 + 26x^2 - 40x + 25 \, dx$$
$$= \pi \left[\frac{x^5}{5} - 2x^4 + \frac{26}{3}x^3 - 20x^2 + 25x \right]_1^4 = \pi \left[\frac{412}{15} - \frac{178}{15} \right] = \frac{234\pi}{15}$$

Ex 2.8: Find vol of solid of rev obtained by rotating region bdd by $y=\sqrt{x}$ + x-axis on $[1,4]$ about x-axis.

Soln:

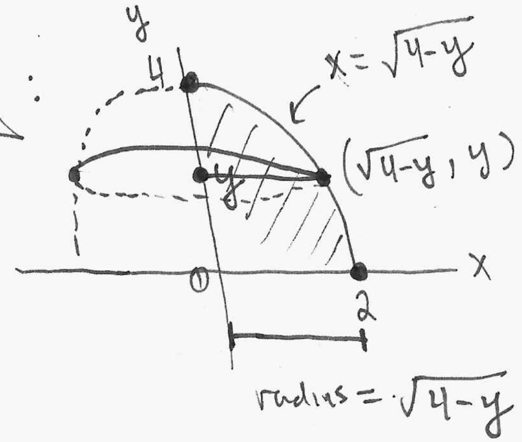


$$\text{Vol} = \pi \int_1^4 (\sqrt{x})^2\,dx = \pi \int_1^4 x\,dx = \pi \frac{x^2}{2}\Big|_1^4$$

$$= \pi\left[\frac{4^2}{2} - \frac{1^2}{2}\right] = \frac{15\pi}{2}$$

Ex: Let R be region bdd by $x=\sqrt{4-y}$ & y-axis on y-axis interval [0,4]. Find vol of solid obtained by rotating R about y-axis.

Soln:



4 y y

$$\text{Vol} = \pi \int_0^4 \left(\sqrt{4-y}\right)^2 dy = \pi \int_0^4 4-y \, dy$$

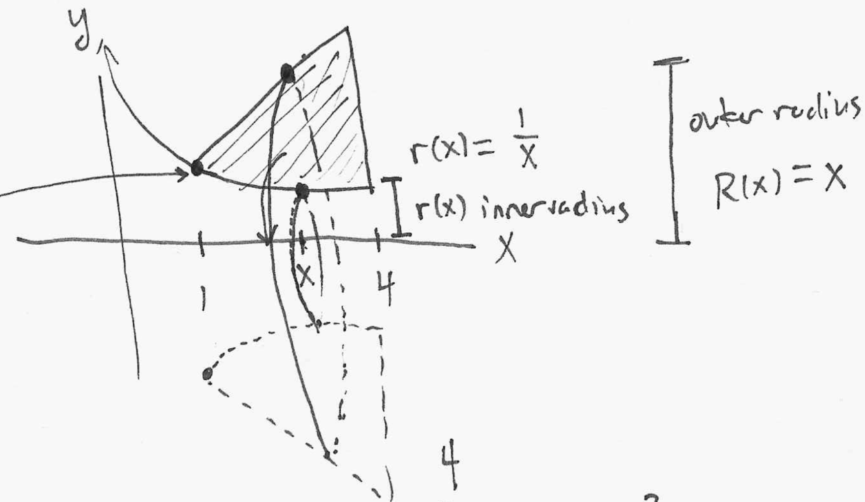
$$= \pi \left[4y - \frac{y^2}{2} \right|_0^4$$

$$= \pi \left[\left(16 - \frac{16}{2}\right) - 0 \right]$$

$$= \frac{16\pi}{2} = 8\pi$$

$\underline{Ex}$: Rotate region bdd ~~by~~ above by $y = x$ & below $y = \frac{1}{x}$ on $[1,4]$ about x-axis.

$\underline{\text{Soln}}$:



$\underline{\text{find} \cap \text{pt}}$

$\frac{1}{x} = x$

$1 = x^2$

$x = \pm\sqrt{1} = \pm 1$

$$\text{Volume} = \pi \int^{4} x^2 - \left(\frac{1}{x}\right)^2 dx$$

$$= \pi \int_1^4 x^2 - x^{-2}\, dx$$

$$= \pi \left[\frac{x^3}{3} - \frac{x^{-1}}{(-1)} \right|_1^4$$

$$= \pi \left[\left(\frac{4^3}{3} + \frac{1}{4} \right) - \left(\frac{1}{3} + 1 \right) \right]$$

$$= \pi \left[\frac{64}{3} + \frac{1}{4} - \frac{1}{3} - 1 \right]$$

$$= \pi \left[\frac{256}{12} + \frac{3}{12} - \frac{4}{12} - \frac{12}{12} \right]$$

$$= \frac{243\pi}{12}$$

$$= \frac{81\pi}{4}$$