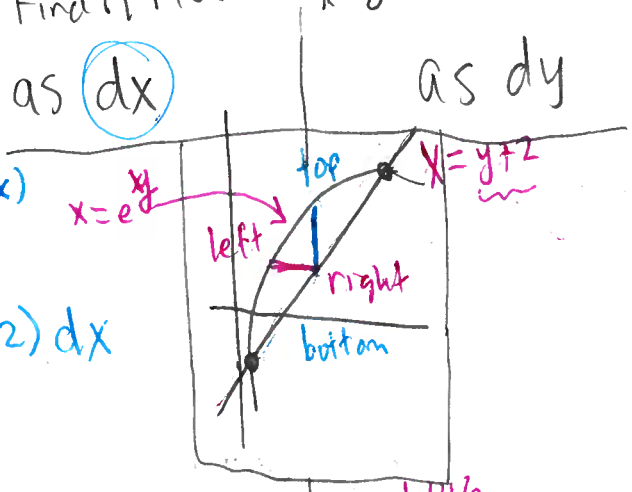
Ex: Find area of region bdd by $x = e^y$ and $y = x - 2$.

Soln: Find ∩ pts: $x = y + 2$ ⇒ set = : $e^y = x = y + 2$

$e^y = y + 2$ → best we can do is numerical approx

Desmos: $y = -1.841, 1.146$

$x = 0.159, 3.146$




3.146

$$\int_{0.159}^{3.146} \ln(x) - (x-2)\,dx$$

What is antideriv of $\ln(x)$?

Answer: $x\ln(x) - x$

Check: $\frac{d}{dx}[x\ln(x) - x]$

$= 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1$

$= \ln(x)$

$$= x\ln(x) - x - \frac{x^2}{2} + 2x \Big|_{0.159}^{3.146}$$

$= 1.803 - (-0.146)$

$= 1.949$

1.146

$$\int_{-1.841}^{1.146} y + 2 - e^y\,dy$$

$$= \frac{y^2}{2} + 2y - e^y \Big|_{-1.841}^{1.146}$$

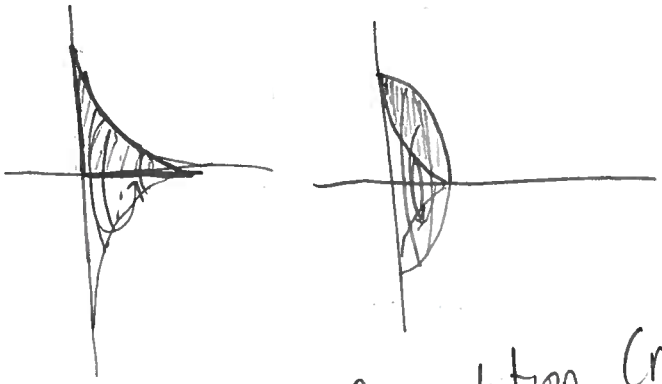
$$= \left(\frac{(1.146)^2}{2} + 2(1.146) - e^{1.146}\right) - \left(\frac{(-1.1841)^2}{2} + 2(-1.841) - e^{-1.846}\right)$$

$\approx -0.197 - (-2.146)$

$= 1.949$

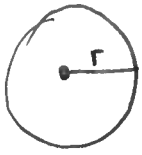
# Solid of Revolution

idea: you have some flat object
and you rotate the object
around some axis (could be x- or y-axis
OR some other line)

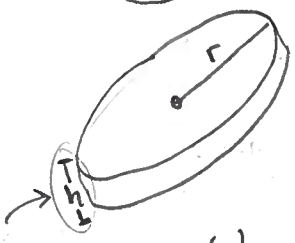


## Volumes of solids of revolution (no holes) (rotate x-axis)

basis:

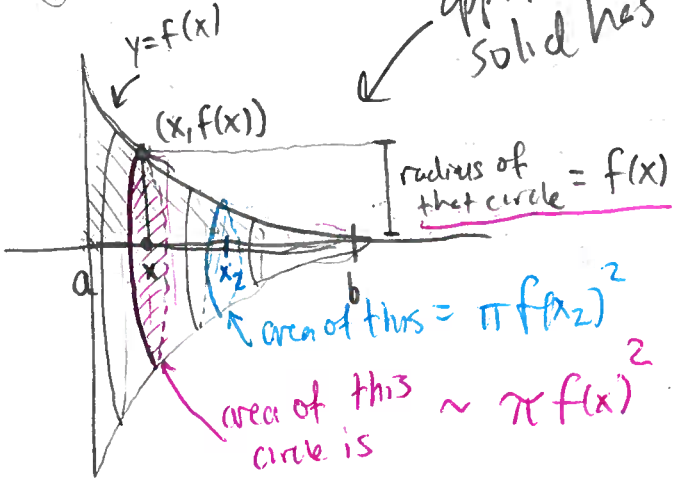



Area $= \pi r^2$




Volume $= \pi r^2 h$

applies when solid has no hole

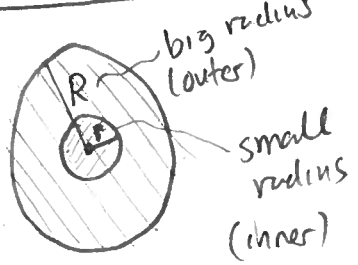



"disk method"

$$\text{Vol} = \int_a^b \pi f(x)^2 \, dx$$

$$= \pi \int_a^b f(x)^2 \, dx$$

# Volumes (with hole) (rotate x-axis)

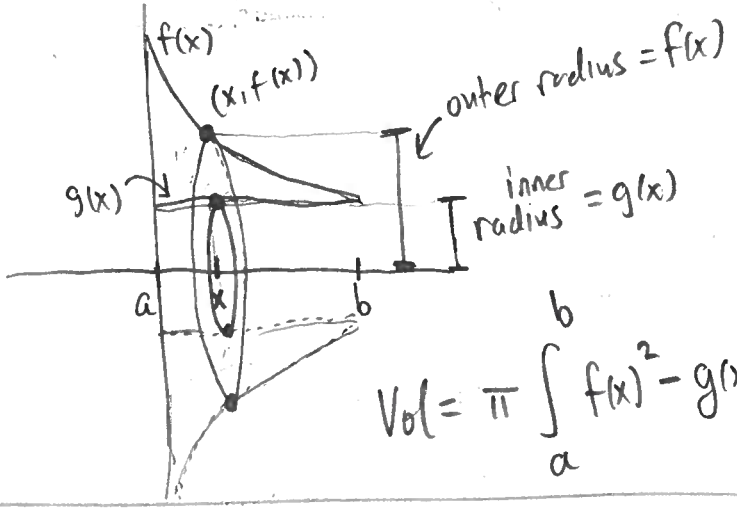



Annulus (aka ring)

area of outer circle $\rightarrow R^2$ , area of inner circle $\rightarrow r^2$

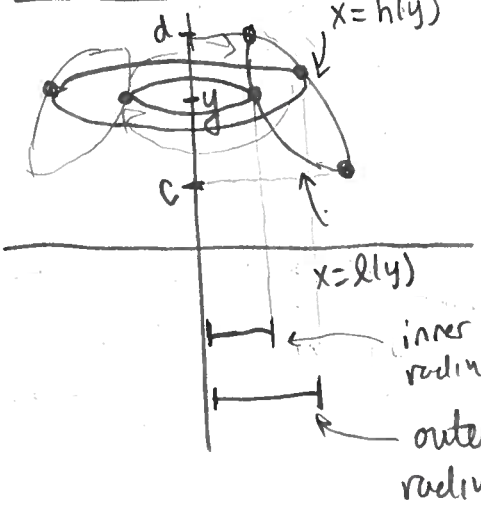
$$\text{Area annulus} = \pi R^2 - \pi r^2$$

$$= \pi[R^2 - r^2]$$




$$\text{Vol} = \pi \int_a^b f(x)^2 - g(x)^2 \, dx$$

# Volumes rotate y-axis




$$\text{Vol} = \pi \int_c^d h(y)^2 - \ell(y)^2 \, dy$$