

find this area

$y = 2 - x$

find this pt

$$x^2 = 2 - x$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$x = -2$  (not relevant)  
 $x = 1$  (good)

Soln:  $\int$  top - bot

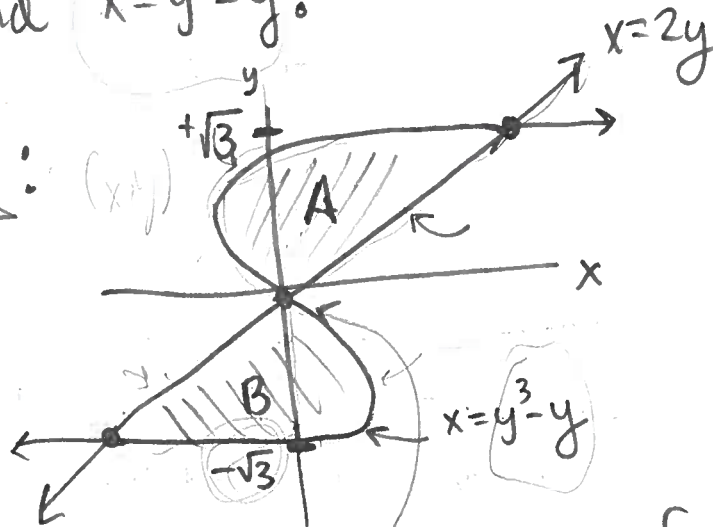
$$\begin{aligned} \text{Area} &= \int_0^1 (2-x) - x^2 dx \\ &= 2x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1 \\ &= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - (0) \\ &= \frac{12}{6} - \frac{3}{6} - \frac{2}{6} \\ &= \frac{7}{6} \end{aligned}$$

(2)

Ex: Find area of region bdd by  $x=2y$

and  $x=y^3-y$ .

Solu:



$\sqrt{\quad}$

∫ right-left dy

Find  $\cap$  pts

$$y^3 - y = 2y$$

$$y^3 - 3y = 0 \rightarrow y(y^2 - 3) = 0$$

$$-y - 2y = -3y$$

$$y = 0$$

OR

$$y^2 - 3 = 0$$

$$y = \pm\sqrt{3}$$

$$\text{Area (A)} = \int_0^{\sqrt{3}} (2y - (y^3 - y)) dy = \int_0^{\sqrt{3}} (3y - y^3) dy = \left[ \frac{3}{2}y^2 - \frac{y^4}{4} \right]_0^{\sqrt{3}}$$

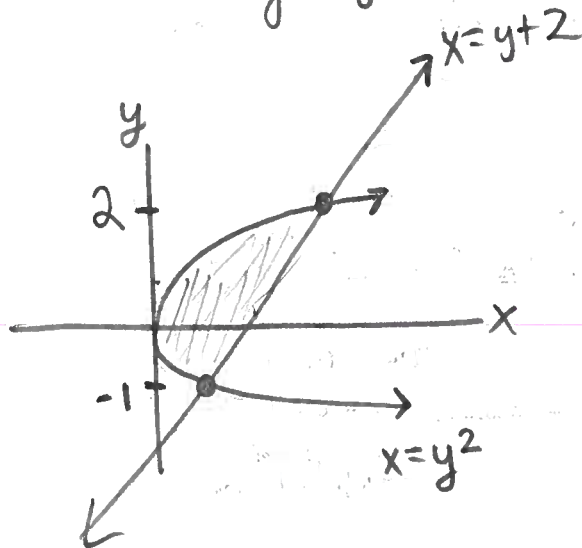
$$\text{Area (B)} = \int_{-\sqrt{3}}^0 ((y^3 - y) - 2y) dy = \int_{-\sqrt{3}}^0 (y^3 - 3y) dy = \left[ \frac{y^4}{4} - \frac{3}{2}y^2 \right]_{-\sqrt{3}}^0$$

$$= \left[ \frac{y^4}{4} - \frac{3}{2}y^2 \right]_{-\sqrt{3}}^0 = 0 - \left( \frac{(-\sqrt{3})^4}{4} - \frac{3}{2}(-\sqrt{3})^2 \right) = -\left( \frac{9}{4} - \frac{9}{2} \right) = -\left( -\frac{9}{4} \right) = \frac{9}{4}$$

$$\text{Area} = \text{Area (A)} + \text{Area (B)} = \frac{9}{4} + \frac{9}{4} = \frac{9}{2}$$

EX: Find area bdd by  $y^2 = x$  and  $x = y + 2$ .

Soln:



Find  $\cap$  pts

$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y - 2)(y + 1) = 0$$

$$y = 2, -1$$

$$\text{Area} = \int_{-1}^2 (y + 2 - y^2) dy$$

$$= \left[ \frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^2$$

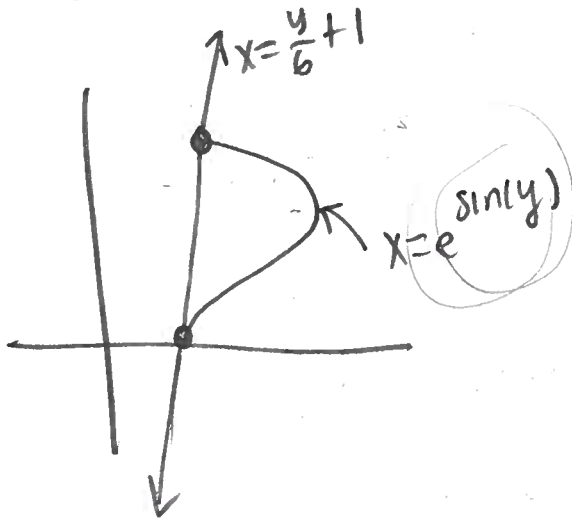
$$= \left[ \frac{4}{2} + 4 - \frac{8}{3} \right] - \left[ \frac{1}{2} - 2 + \frac{1}{3} \right]$$

$$= \left[ \frac{18}{3} - \frac{8}{3} \right] - \left[ \frac{1}{2} - \frac{6}{3} + \frac{1}{3} \right]$$

$$= \frac{20}{6} - \left[ \frac{3}{6} - \frac{12}{6} + \frac{2}{6} \right]$$

$$= \frac{7}{6}$$

Ex: From desmos

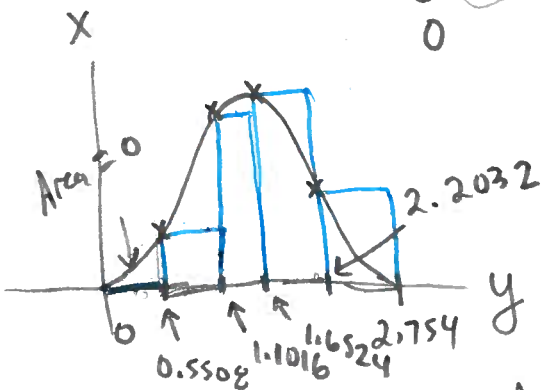


Find  $n$  pts

$\left(\frac{y}{6} + 1 = e^{\sin(y)}\right) \leftarrow$  can we solve?  
 $\uparrow$   
 not symbolically

$\Rightarrow$  graph:  $n$  pts are at  
 $y=0$  and  $y=2.754$

Area =  $\int_0^{2.754} (e^{\sin(y)} - (\frac{y}{6} + 1)) dy \leftarrow$  can you find antideriv?  
 NO



Cut region into  
 $5$  rect

$$\Delta x = \frac{2.754 - 0}{5} \approx 0.5508$$

$$\text{Area Estimate} = 0 + 0.5508 [0.5959 + 1.2564 + 1.4338 + 0.000305]$$

$$\approx 1.81$$