$$\int \frac{5}{\sqrt{9x^2+6x-8}}\,dx$$

problem

$$9x^2+6x-8 = 9\left[x^2+\frac{6}{9}x-\frac{8}{9}\right]$$

$$\frac{1}{\sqrt{9x^2+6x-8}} = \frac{1}{\sqrt{9\left[x^2+\frac{6}{9}x-\frac{8}{9}\right]}}$$

$$= \frac{1}{3}\,\frac{1}{\sqrt{x^2+\frac{2}{3}x-\frac{8}{9}}}$$

$b=\frac{2}{3} \rightsquigarrow$ complete square

$$\Rightarrow \int \frac{5}{\sqrt{9x^2+6x-8}}\,dx = \frac{5}{3}\int \frac{1}{\sqrt{x^2+\frac{2}{3}x-\frac{8}{9}}}\,dx$$

$$x^2+\frac{2}{3}x-\frac{8}{9} = \left(x+\frac{b}{2}\right)^2-\frac{8}{9}-\left(\frac{b}{2}\right)^2$$

$b=\frac{2}{3}$

$$= \left(x+\frac{1}{3}\right)^2-\frac{8}{9}-\frac{1}{9}$$

$\frac{b}{2}=\frac{1}{3}$

$$= \left(x+\frac{1}{3}\right)^2-1$$

$$\Rightarrow \int \frac{5}{\sqrt{9x^2+6x-8}}\,dx = \frac{5}{3}\int \underbrace{\frac{1}{\sqrt{(x+\frac{1}{3})^2-1}}\,dx}_{\text{what is this?}}$$

from Wikipedia

$$\boxed{\operatorname{arcosh}(x) = \int \frac{1}{\sqrt{x^2-1}}\,dx}$$

$u = x+\frac{1}{3}$

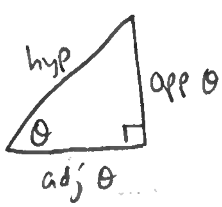
$du = dx$

$$= \frac{5}{3}\int \frac{1}{\sqrt{u^2-1}}\,du$$

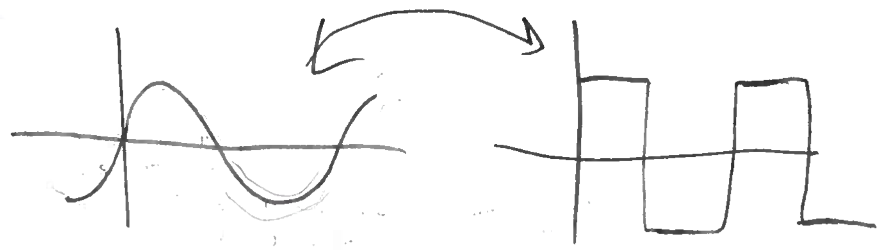
NOT arcsin

$$= \frac{5}{3}\operatorname{arccosh}(u)+C$$

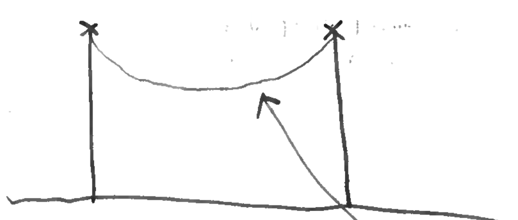
$$= \frac{5}{3}\operatorname{arccosh}\left(x+\frac{1}{3}\right)+C$$



$\sin\theta = \frac{\text{opp }\theta}{\text{hyp}}$



hyperbolic trig function



not a parabola ~ "catenary" curve

Hyperbolic trig functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

"sinch"

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\frac{d}{dx}\sinh(x) = \frac{d}{dx}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{1}{2}\frac{d}{dx}\left[e^x - e^{-x}\right]$$

$$= \frac{1}{2}\left[e^x + e^{-x}\right]$$

$$= \cosh(x)$$

Similarly,

$$\frac{d}{dx}\cosh(x) = \frac{d}{dx}\left[\frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{1}{2}\left[e^x - e^{-x}\right]$$

$$= \sinh(x)$$

(4)

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$y = 2x + 1$$

$$\frac{y-1}{2} = x$$

$$f(x) = \frac{e^x + e^{-x}}{2}$$

$$f(x) = 2x + 1$$

$$\boxed{f^{-1}(x) = ??}$$

$$f^{-1}(x) = \frac{x-1}{2}$$

$$a^{-1} = \frac{1}{a}$$

$$y = \frac{e^x + e^{-x}}{2}$$

$$= \frac{e^x + \frac{1}{e^x}}{2}$$

$$= \frac{e^{2x} + 1}{2e^x}$$

$$\cancel{(e^x + 1)^2 - 2e^x}$$

$$\cancel{= e^{2x} + 2e^x + 1 - 2e^x}$$

$$\cancel{= e^{2x} + 1}$$

$$ax^2 + bx + c = 0$$

$$\downarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2ye^x = e^{2x} + 1$$

$$0 = e^{2x} - 2ye^x + 1 \leftarrow \text{"quadratic in form"}$$

$$e^{2x} = (e^x)^2$$

New var: $w = e^x$

$$w^2 = e^{2x}$$

$$0 = w^2 - 2yw + 1 \leftarrow \text{quadratic!!}$$

$$\downarrow \text{QF}$$

$$w = \frac{\cancel{2}y \pm \sqrt{\cancel{4}y^2 - \cancel{4}}}{\cancel{2}} = y \pm \sqrt{y^2 - 1}$$

$\pm$ depends on which half of cosh we are inverting

$$e^x = y \pm \sqrt{y^2 - 1} \xrightarrow{\log} x = \ln\left(y \pm \sqrt{y^2 - 1}\right)$$

Trig functions

$i = \sqrt{-1}$

Euler's formula: $\boxed{e^{ix} = \cos(x) + i\sin(x)}$

ex) $e^{i\pi} = \underbrace{\cos(\pi)}_{=-1} + i\underbrace{\sin(\pi)}_{=0}$

$e^{i\pi} = -1$

$e^{-ix} = \cos(-x) + i\sin(-x)$

$\underbrace{\cos(-x)}$: cos is even $= \cos(x)$

$\underbrace{\sin(-x)}$: sin odd $= -\sin(x)$

$e^{ix} = \cos(x) + i\sin(x)$

$e^{-ix} = \cos(x) - i\sin(x)$

add

$e^{ix} + e^{-ix} = 2\cos(x)$

$\rightarrow \cos(x) = \dfrac{e^{ix} + e^{-ix}}{2}$

subtract

$e^{ix} - e^{-ix} = 2i\sin(x)$

$\sin(x) = \dfrac{e^{ix} - e^{-ix}}{2i}$