2021.01.19

$\frac{d}{dt}\ln(t) = \frac{1}{t}$ $\qquad \frac{d}{d☺}\ln(☺) = \frac{1}{☺}$

$\frac{d}{dx}\cos(x) = -\sin(x)$

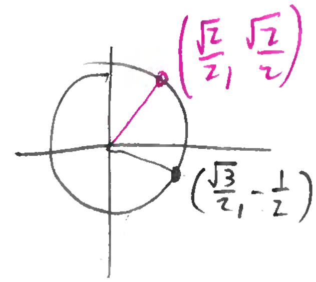
Ex:

$$\int \frac{(\ln(x))^{31}}{x}\,dx = \int u^{31}\,du = \frac{u^{32}}{32} + C = \frac{(\ln(x))^{32}}{32} + C$$

$u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$



Ex:

$$\int_{w=-\pi/6}^{w=\pi/4} \cos^3(w)\sin(w)\,dw = \int_{\sqrt{3}/2}^{\sqrt{2}/2} \alpha^3(-1)\,d\alpha$$

$\alpha = \cos(w)$ $\qquad w = -\frac{\pi}{6} \rightarrow \alpha = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\frac{d\alpha}{dw} = -\sin(w)$ $\qquad w = \frac{\pi}{4} \rightarrow \alpha = \cos(\pi/4) = \frac{\sqrt{2}}{2}$

$d\alpha = -\sin(w)\,dw$

$-d\alpha = \sin(w)\,dw$

$$= -\frac{\alpha^4}{4}\Bigg|_{\alpha=\sqrt{3}/2}^{\alpha=\sqrt{2}/2}$$

$$= \left[-\frac{(\sqrt{2}/2)^4}{4} + \frac{(\sqrt{3}/2)^4}{4}\right]$$

$2 \cdot 16 \quad 4 \quad 64$ $\qquad (\sqrt{2})^4 = 2 \cdot 2 = 4$

$$= -\frac{4}{64} + \frac{9}{64}$$

$$= +\frac{5}{64}$$

Ex: (unnormalized) Fresnel integral

$$C(x) = \int_{t=0}^{t=x} \cos(t^2)\,dt \longrightarrow C(\sqrt{7}x^2) = \int_0^{\sqrt{7}x^2} \cos(t^2)\,dt$$

Q: Evaluate $\int_0^x t\cos(7t^4)\,dt$ in terms of C.

insight: pick a variable u such that $u^2 = 7t^4$

$$\Updownarrow \sqrt{}$$

$u = \sqrt{7}t^2$

$du = 2\sqrt{7}\,t\,dt$

$\frac{du}{2\sqrt{7}} = t\,dt$

if $t=0$, then $u = 7(0^4) = 0$

if $t=x$, then $u = \sqrt{7}x^2$

Compute

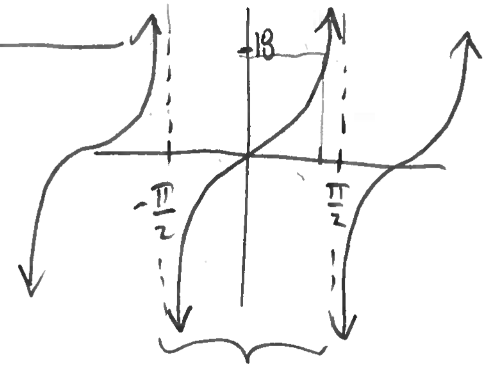
$$\int_0^x t\cos(7t^4)\,dt = \int_0^{\sqrt{7}x^2} \cos(u^2)\frac{1}{2\sqrt{7}}\,du$$

$$= \frac{1}{2\sqrt{7}} \boxed{\int_0^{\sqrt{7}x^2} \cos(u^2)\,du}$$

write in terms of C

$$= \frac{1}{2\sqrt{7}} C(\sqrt{7}x^2)$$

Integrals involving inverse trig functions

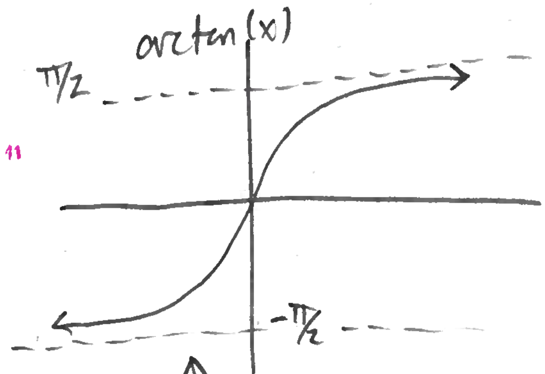


Q: What is $\frac{d}{dx}\arctan(x)$

Recall: chain rule says

$$\frac{d}{dw} f(t) = \frac{dt}{dw}\frac{d}{dt} f(t)$$

"sort of = 1"



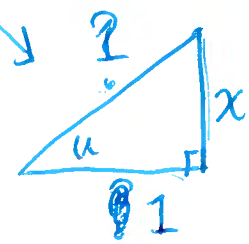
since arctan is inverse of tan ~

$\tan(\arctan x) = x$

A: $u = \arctan(x)$ $\quad \frac{du}{dx}$

plug both sides into tangent

$$\boxed{\tan(u) = \tan(\arctan(x)) = x}$$



$x = \frac{x}{1}$ ← opp, adj

$1^2 + x^2 = ?^2$

$? = \sqrt{x^2+1}$

take $\frac{d}{dx}$ of both sides

$$\frac{d}{dx}\tan(u) = \frac{d}{dx}[x] = 1$$

different → ALWAYS chain rule

chain rule

$$\frac{du}{dx}\frac{d}{du}\tan(u) = 1 \longrightarrow \frac{du}{dx}\frac{1}{\cos^2(u)} = 1$$

Solve for this

$$= \frac{d}{du}\frac{\sin u}{\cos u} = \frac{\cos^2(u) + \sin^2(u)}{\cos^2(u)} = \frac{1}{\cos^2(u)}$$

$$\frac{du}{dx} = \cos^2(u) = \left(\frac{adj}{hyp}\right)^2 = \left(\frac{1}{\sqrt{x^2+1}}\right)^2 = \frac{1}{1+x^2}$$