2021.01.15

$$e^{\ln(x)} = x$$

$$\ln(e^{x}) = x$$

Ex: $\displaystyle\int_{x=e^2}^{x=e^5} \frac{1}{x\ln(x)}\,dx = \int_{u=2}^{u=5} \frac{1}{u}\,du$

define u $\rightarrow$ $\boxed{u = \ln(x)}$ if $x = e^2$, then $u = \cancel{\ln}(\cancel{e}^2) = 2$

$du = \frac{1}{x}\,dx$ if $x = e^5$, then $u = \cancel{\ln}(\cancel{e}^5) = 5$

$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$

$\int dt$

$$= \ln(u)\Big|_{u=2}^{u=5} = \ln(5) - \ln(2)$$

$$= \ln\left(\frac{5}{2}\right)$$

a number!

An "alt" approach

$$\int_{e^2}^{e^5} \frac{1}{x\ln(x)}\,dx = \int_{x=e^2}^{x=e^5} \frac{1}{u}\,du = \ln(u)\Big|_{x=e^2}^{x=e^5}$$

$$= \ln(\ln(x))\Big|_{x=e^2}^{x=e^5}$$

$$= \ln(\cancel{\ln}(\cancel{e}^5)) - \ln(\ln(e^2))$$

$$= \ln(5) - \ln(2)$$

2

$x^{\frac{a}{b}} = \sqrt[b]{x^a}$

$x^{-1} = \frac{1}{x}$

Ex: $\int_0^1 \frac{5x^2}{\sqrt{2-x^3}}\,dx = \int_2^1 \frac{5}{\sqrt{u}}\left(-\frac{1}{3}\right)du$

$\int_a^b = -\int_b^a$

$\boxed{u = 2 - x^3}$

$du = -3x^2\,dx$

$\frac{du}{-3} = x^2\,dx$

if $x=0 \rightarrow u = 2-0^3 = 2$

if $x=1 \rightarrow u = 2-1^3 = 1$

$= -\int_1^2 \frac{5}{\sqrt{u}}\left(-\frac{1}{3}\right)du$

constant

$= \frac{5}{3}\int_1^2 u^{-1/2}\,du$

$= \frac{5}{3}\left.\frac{u^{1/2}}{1/2}\right|_1^2$

$= \frac{10}{3}\left[2^{1/2} - 1^{1/2}\right]$

$= \frac{10}{3}\left[\sqrt{2} - 1\right]$

Ex: $\int \frac{\cos(\ln(x))}{x}\,dx = \int \cos(u)\,du$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \sin(u) + C$

$= \sin(\ln(x)) + C$

Like written HW: error function

$$\boxed{\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt}$$

TRY

$u = -5t^2$

Evaluate $\int_0^x e^{-5t^2}\,dt$ in terms of erf. $du = -10t\,dt$

no t in our problem

won't work

$W = \sqrt{5}\,t$

square both sides

$w^2 = (\sqrt{5}t)^2 = 5t^2$

$dw = \sqrt{5}\,dt$

$\frac{dw}{\sqrt{5}} = dt$

if $t=0 \to w=0$

if $t=x \to w=\sqrt{5}\,x$

$$= \int_0^{\sqrt{5}x} e^{-w^2}\frac{1}{\sqrt{5}}\,dw$$

$$= \frac{1}{\sqrt{5}}\boxed{\int_0^{\sqrt{5}x} e^{-w^2}\,dw}$$

$$\frac{\sqrt{\pi}}{2}\mathrm{erf}(x) = \int_0^x e^{-t^2}\,dt$$

$\downarrow x \mapsto \sqrt{5}\,x$

$$\frac{\sqrt{\pi}}{2}\mathrm{erf}(\sqrt{5}\,x) = \boxed{\int_0^{\sqrt{5}x} e^{-t^2}\,dt}$$

SAME

$$= \frac{1}{\sqrt{5}}\frac{\sqrt{\pi}}{2}\mathrm{erf}(\sqrt{5}\,x)$$