Ex: $\int \cos(t)\sin^6(t)\,dt = \int u^6\,du = \frac{u^7}{7}+C$

$\int \cos t\,dt = \sin(t)+C$

$\int \sin t\,dt = -\cos(t)+C$

$u=\sin(t)$ $\quad u^6=\sin^6(t)$ $\quad = \frac{\sin^7(t)}{7}+C$

$\frac{du}{dt}=\cos(t)$

$du=\cos(t)\,dt$

You have to decide what u is

What if we picked a different change of variable?

$w=\cos t$ $\qquad \int \cos(t)\sin^6(t)\,dt$

$\frac{dw}{dt}=-\sin t$

$dw=-\sin t\,dt$

can't take powers here

$dt=\frac{dw}{-\sin t}$

$=\int w$ how to write $\sin^6(t)$ in terms of w? how to handle dt?

$=\int w\,(\sin^6(t))?\,\frac{dw}{\sin(t)}$

Ex: $\int \frac{\boxed{e^t+8 \; dt}}{e^t+8t}$

$\int \frac{e^t+8}{e^t+8}\,dt$ $\quad \frac{d}{dt}t^2=2t$

$u=e^t+8t$

$\int \frac{du}{u}$

$w=e^t+8$

$v=e^t$

Do u

$u=e^t+8t$

$du=\boxed{e^t+8\ dt}$

parts of integral

the answer to the integral

$$\int \frac{e^t+8}{e^t+8t}\,dt=\int \frac{1}{u}\,du=\ln(u)+C$$

$$=\ln(e^t+8t)+C$$

Do w

$w=e^t+8$

$dw=e^t\,dt$

$dt=\frac{dw}{e^t}=\frac{dw}{w-8}$

$e^t=w-8$

$$\int \frac{e^t+8}{e^t+8t}\,dt=\int \frac{w}{?}\,\frac{1}{w-8}\,dw$$

how to express e^t+8t in terms of w? CAN'T!

Do ✓

$\ln(v)=t$

$v=e^t$

$dv=e^t\,dt$

$dt=\frac{dv}{e^t}=\frac{1}{v}\,dv$

how to solve? This is a more difficult problem.

$$\int \frac{e^t+8}{e^t+8t}\,dt=\int \frac{v+8}{v+8\ln(v)}\,\frac{1}{v}\,dv$$

Ex: $\int 2x^2\sqrt{1+x^3}\,dx$

$u=1+x^3$

$du=3x^2\,dx$

$\frac{du}{3}=x^2\,dx$

$=\int 2\,\frac{1}{3}\sqrt{u}\;du$

$=\frac{2}{3}\int u^{1/2}\,du$

$=\frac{2}{3}\,\frac{u^{3/2}}{3/2}+C$

$=\frac{4}{9}u^{3/2}+C$

$=\frac{4}{9}(1+x^3)^{3/2}+C$

Recall

$\sqrt{x}=x^{1/2}$

$\int\sqrt{x}\,dx=\int x^{1/2}dx$

$=\frac{x^{1/2+1}}{1/2+1}+C$

$=\frac{x^{3/2}}{3/2}+C$

$=\frac{2x^{3/2}}{3}+C$

$\sec(x) = \frac{1}{\cos x}$

Ex: $\int \tan(x)\,dx = \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

$= -\int \frac{1}{u}\,du$

$= -\ln(u) + C$

$= -\ln(\cos(x)) + C$

careful to never plug in x where $\cos(x) < 0$