Fundamental Theorem of Calculus

1. $\dfrac{d}{dx}\displaystyle\int_a^{x} f(t)\,dt = f(x)$

t is a "dummy variable"

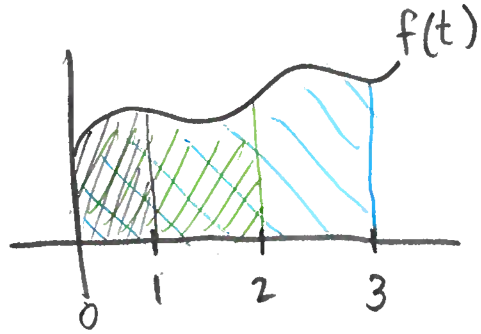
Think about $F(x) = \displaystyle\int_0^x f(t)\,dt$ as an "area function"

$F(0) = \displaystyle\int_0^0 f(t)\,dt = 0$

$F(1) = \displaystyle\int_0^1 f(t)\,dt$

$F(2) = \displaystyle\int_0^2 f(t)\,dt$

$F(3) = \displaystyle\int_0^3 f(t)\,dt$



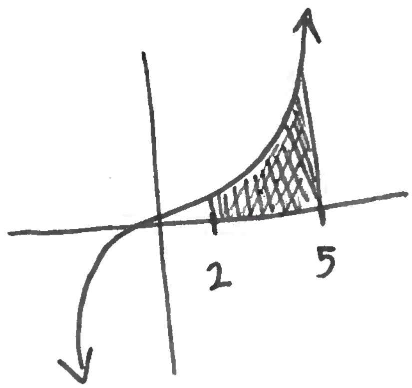
2. $\displaystyle\int_a^b f'(t)\,dt = D_t^{-1} f'(t)\Big|_a^b = f(t) + C\Big|_{t=a}^{t=b}$

$= (f(b) + \cancel{C}) - (f(a) + \cancel{C})$

$= f(b) - f(a)$

$f'(t) = \dfrac{df}{dt}$

Ex: $\displaystyle\int_2^5 t^3\,dt = \left.\frac{t^4}{4}\right|_{t=2}^{t=5} = \frac{5^4}{4} - \frac{2^4}{4}$



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sss
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s

$-2 = -2\underbrace{s^0}_{=1}$

Ex: $\displaystyle\int_{-1}^{3} s^2+3s-2\,ds = \left.\frac{s^3}{3}+\frac{3s^2}{2}-2s\right|_{s=-1}^{s=3}$

$$= \left[\frac{3^3}{3}+\frac{3\cdot 3^2}{2}-2(3)\right] - \left[\frac{(-1)^3}{3}+\frac{3(-1)^2}{2}-2(-1)\right]$$

$$= \dots$$

$D_w^{-1}(e^w) = e^w + C$

Ex: $\displaystyle\int_{10}^{15} e^w\,dw = \left.e^w\right|_{w=10}^{w=15} = e^{15} - e^{10}$

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x

Ex: $\displaystyle\int_{107}^{6124} \frac{1}{\xi}\,d\xi = \left.\ln(\xi)\right|_{\xi=107}^{\xi=6124} = \ln(6124) - \ln(107)$

$\log(\xi)$ ok

§1.5 Anti-Chain Rule (Substitution or change of variables)

Recall chain rule for derivatives:

$$\frac{d}{dt} f(g(t)) = \frac{df}{dg}\frac{dg}{dt} = f'(g(t))g'(t)$$

$\int_a^b$ both sides

Thm 1.7

$$D^{-1}[\underline{f'(g(x))}\underline{g'(x)}] = f(g(x))+C$$

$$\underbrace{\int_a^b \frac{d}{dt} f(g(t))dt}_{\text{can be done w/ FTC}} = \int_a^b f'(g(t))g'(t)\,dt$$

this is where you usually start

$$= f(g(t))\Big|_a^b$$

$$= f(t)\Big|_{g(a)}^{g(b)}$$

Ex: $D_x^{-1}\left[6x(3x^2+4)^4\right]$

without substitution:

$$D_x^{-1}[\underbrace{6x(3x^2+4)^4}_{\text{expand fully}}] = D_x^{-1}[486x^9+2592x^7+5184x^5+4608x^3+1536x]$$

$$= \frac{486x^{10}}{10}+\frac{2592x^8}{8}+\frac{5184x^6}{6}+\frac{4608x^4}{4}+\frac{1536x^2}{2}+C$$

with substitution

Thm 1.7

$$D_x^{-1}\left[6x(3x^2+4)^4\right] = f(g(x)) + C = \frac{(3x^2+4)^5}{5} + C$$

if you write

$f'(x) = x^4 \rightarrow \boxed{f(x) = D_x^{-1}[x^4] = \frac{x^5}{5} + C}$

$\boxed{g(x) = 3x^2+4} \rightarrow g'(x) = 6x$

$f'(g(x)) = (3x^2+4)^4$

Indefinite integral symbol

instead of $D_x^{-1}[f(x)]$

we will write $\int f(x)\,dx$

no bounds

$$\int 6x(3x^2+4)^4\,dx = \int u^4\,du = \frac{u^5}{5} + C$$

$u = 3x^2+4$

$\frac{du}{dx} = 6x$

$du = 6x\,dx$

$$= \frac{(3x^2+4)^5}{5} + C$$