Review of calc 1

$$\frac{d}{dx}x^n = nx^{n-1} \; ; \; n = 1, 2, \ldots$$

$$\boxed{\frac{d}{dx}e^{x} = e^{x}}$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

log

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin(x)$$

Product Rule: $\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

Chain Rule:

$$\rightarrow \frac{d}{dt}e^{5t} = \frac{d(5t)}{dt}\boxed{\frac{d}{d(5t)}e^{5t}} = 5e^{5t}$$

$$\rightarrow \frac{d}{dt}f(g(t)) = \frac{df}{dg}\frac{dg}{dt} = f'(g(t))g'(t)$$

Integration

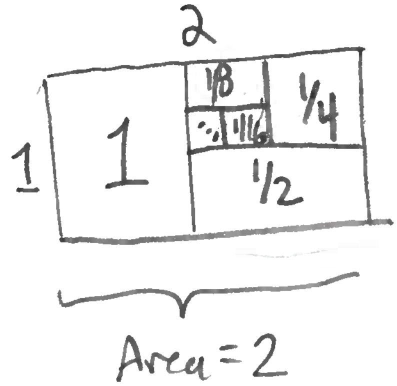
Recall sum notation $\sum$

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \dots + a_n$$

$$\sum_{i=1}^{n} i = 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

later: $$\sum_{i=0}^{\infty} \frac{1}{2^i} = 2$$

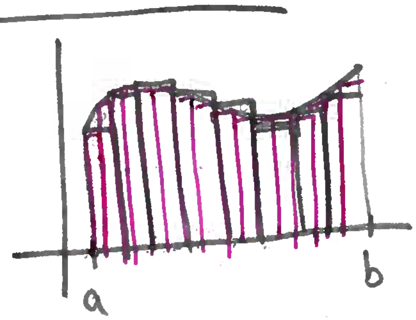
$$= 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$$



$$\sum_{i=1}^{\infty} \frac{1}{i^2} = 1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\dots = \frac{\pi^2}{6}$$

Definite integration ~ "area under curve"

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \overbrace{f(x_i)}^{\text{height}}\,\overbrace{\Delta x}^{\text{width}}$$



where $\Delta x = \frac{b-a}{n}$

and $x_1 = a$

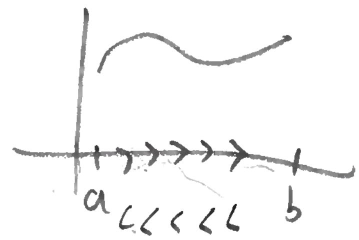
$x_n = b$

$x_{i+1} = x_i + \Delta x$

Properties of $\int_a^b f(x)\,dx$

① $\int_a^a f(x)\,dx = 0$

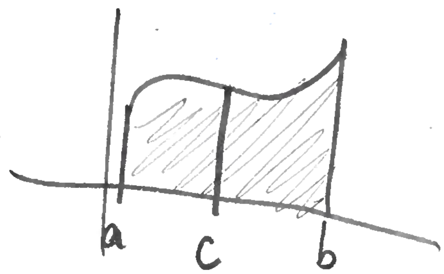
② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$



③ $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

④ $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

⑤ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$



Antiderivatives

An antiderivative of a function f is a function F such that the derivative of F is f ~ $\frac{d}{dx}F(x) = f(x)$.

We use D_x^{-1} to denote antiderivative with respect to x.

All derivative rules have corresponding antiderivative rule.

Power rule (derivs)

$$\frac{d}{dx} x^n = n x^{n-1}$$

(antideriv)

$$x^n = D_x^{-1}(n x^{n-1})$$

$$D^{-1}(x^{n-1}) = \frac{x^n}{n} + C$$

$$D_x^{-1}(x^n) = \frac{x^{n+1}}{n+1} + C$$

Ex: $D_x^{-1}(x^3) = \frac{x^4}{4} + C$

arbitrary constant

$$D_t^{-1}(t^4) = \frac{t^5}{5} + C$$

$$D_{☺}^{-1}(☺^8) = \frac{☺^9}{9} + C$$