Written HW9 – MATH 2502 Fall 2021

Due by 16 September for timely completion credit

Consider the curve $f(t) = \dfrac{1}{t}$ on the interval $[1, x]$, where $x > 1$ is some number (*note: you don't pick x – use "x" in your calculations*).

#1. Sketch the curve and its shadow region.

#2. Find the volume of the solid obtained by taking the region defined on top by $f(t)$ and on the bottom by the horizontal axis and rotating it about the horizontal axis. Use your preferred method to find the volume (*note: the volume will contain the variable x*). All details must be shown when computing the integral.

#3. **Set up but do not evaluate** the integral that computes the surface area obtained when rotating the curve $f(t)$ over $[1, x]$ about the horizontal axis. (*note: the surface area will contain the variable x*).

#4. Use Desmos to plot the integral found in #3 above as a function of x. On the same plot, also plot the function $2\pi \ln(x)$ — which is bigger when $x > 1$? Include your plots in your answer.

#5. Compute the limit of the volume computed in #1 as $x \to \infty$ (this resembles a calculus 1 problem). Use your plots in #4 to conclude what the surface area computed in #2 becomes as $x \to \infty$.

#6. "Gabriel's horn" is the surface of revolution formed by rotating the curve $\dfrac{1}{x}$ lying above the infinite interval $[0, \infty)$ across the x-axis. Based on your answer to #5, what conclusion can you make about the volume and surface area of Gabriel's horn? It is often said that "*Gabriel's horn can be filled with paint, but it can never be painted*" – explain what that means in the context of your calculations.